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WATER RIGHTS AND OPTIMAL RESERVOIR MANAGEMENT

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I. INTRODUCTION

In many areas, notably the arid portion of the western United States, economic development and increased use of both surface and ground water supplies has been accompanied by proliferation of water storage facilities. As these reservoirs are filling, rivers become more completely appropriated and ground water supplies approach depletion more rapidly. While the analysis and management of ground water and surface water supplies should be integrated, we consider surface water independently, thus following the bifurcation in the treatment of ground and surface water under existing water law.** In Burness and Quirk [3] we considered the efficiency aspects of the appropriative water rights doctrine in the static analogy of an uncontrolled river. Here we follow in the spirit of Gessford and Karlin [6] and analyze the optimal operation of a water storage facility in the presence of downstream users. In addition, we consider alterations in diversion capacity by water users in the presence of such a facility. By explicitly considering alternative water rights doctrines we are able to comment on the efficiency and stability of prevalent patterns of ownership and operation.

In the eastern United States water rights have developed according to the English common law doctrine of riparianism. Under the riparian

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**For summary of water law see [12].

doctrine, each property owner fronting on a lake or stream has a right to the unimpaired use of the waterway, regardless of the location of his property along the waterway and regardless of the time at which the property is acquired or use made of the waterways. As all rights are equal, strictly speaking the right holder may not diminish the flow of water by physically consuming it, as this would impair the rights of other riparians. While courts have held that "reasonable" diversions of water by riparian rights holders are permissible, there are still severe restrictions on such diversions, coupled with uncertainty as to the judicial response to any proposed diversion. Due to the common property problems associated with the riparian doctrine and its inappropriateness for water scarce areas, we do not consider it analytically. Rather we focus on the doctrine of prior appropriation and the doctrine of correlative rights.

Under the appropriative doctrine the right to a certain amount of water is established and maintained only through use; depending on state law, rights may be lost by abandonment, forfeiture, prescription or estoppel. Priority of rights is determined by the chronological order in which the rights were obtained, the earliest right being the most senior.*

The doctrine of appropriation suffers from several drawbacks. For example, under this doctrine an individual can appropriate more water than he can presently use in order to provide for his needs in

*For a thorough discussion of the legal aspects of appropriation, see [12].

the future when such excess water might be used profitably.* While state water laws limit appropriations to uses that are "beneficially consumptive" in an attempt to preclude methods or types of use which are wasteful, there are obvious difficulties in establishing that water is being wasted by a rights holder, so that the protection afforded through restriction of appropriations to beneficial consumptive use may be more illusory than real.**

However, even when the tenet of beneficial consumptive use is strictly adhered to, there are still problems with the appropriative system. In [3] we show that the doctrine of appropriation leads to allocative inefficiencies stemming from the unequal sharing of risk among senior and junior appropriators; we also show, as an application of the Coase theorem [4], that the introduction of competitive markets in water rights and the use of diversion facilities eliminates these inefficiencies. While in principle water rights could be freely transferable under the appropriative doctrine, in practice there are limitations on the transfer and sale of water rights. These restrictions appply most forceably to a change in type of use or in diversion locations, as for example, in the transfer of water rights from irrigation to municipal use, or in a transfer of water outside the

*To illustrate, a large western irrigation district loses perhaps 500,000 acre-feet of water yearly through seepage in an unlined diversion canal, a method of use which could be considered wasteful. As the district has established rights to this water, the future lining of the canal would make the salvaged water available to the district.

**However, there are exceptions. Struckmeyer and Butler[11] cite a California court case in which the use of water during the off-season to flood gophers from their holes was considered wasteful.

property limits of the original rights holder.* Moreover, sale or transfer of water involving the removal of water to another state is a practical impossibility, at least in the western states.

Nevertheless, the appropriative doctrine has been widely adopted in the west** as it is well suited to the exploitation of a waterway in a scenario where water is scarce and the major uses of the waterway must involve physical diversions, say for irrigation or for municipal or industrial uses. Under such circumstances there are obvious advantages to a system of rights based on the appropriative doctrine. An appropriative allocation preserves incentives for investment that might be foregone under the riparian scheme due to the common property aspects of the latter. Moreover, while the appropriative allocation is not Pareto optimal in a steady state equilibrium, such a system of rights is necessary for any substantive development to occur at all.

An alternative doctrine to that of appropriation is a variant of the doctrine of correlative water rights from ground water law. The doctrine of correlative rights assigns the owner of land overlying a ground water basin rights to water based on the percentage of his land to all such overlying land. The variant we employ is that of equal sharing,

*Often such transfers alter the availability, quantity or quality of return flows and may impair the rights of previously adjacent appropriators. In 1974, the Metropolitan Water District of Southern California was able to transfer a portion of its rights to Colorado River water to the Southern California Edison Company, but only after the passage of enabling legislation by the California State Legislature, as the So. California Edison intended to use this water outside of the geographic limits of the MWD.

**Actually the appropriative doctrine is spreading to the Eastern states as well, although for the wrong reasons. See Milliman [9].

in the case of identical downstream firms (users), or prorationing, in the case that downstream firms might differ.* While equal sharing is Pareto superior to appropriation (assuming appropriate compensation), equal sharing may preclude optimal investment from being undertaken due to a lack of "tenure certainty"**: the protection of a water right against the lawful acts of others. For this reason we consider the doctrine of equal sharing only when further entry or exit is precluded or when optimal development of the waterway has previously occurred. Certainly a doctrine such as appropriation is essential in arriving at this juncture.

Substantial amounts of stored water are supplied by Bureau of Reclamation or Corps of Engineer projects. Corps projects usually contract storage space to water users, who have obtained rights under state law, and then impound water and make it available for delivery to those individuals. Bureau of Reclamation projects usually acquire the water right directly from the state and then wholesale such water to water districts which in turn retail the water to users with whom they have contracts. This is done at a cost which purports to amortize reimbursable construction costs and costs of operation and maintenance. Irrigation water usually is subsidized, whereas municipal and industrial water repay their full prorated share of the cost. Hydroelectric power sales amortize project costs not accounted for by the sale of water through delivery contracts.

*In this paper we consider only the case of identical downstream users. For an analysis of non-identical and possibly risk-averse firms in a static setting with no storage see [3].

**See Milliman [9].

As we are concerned primarily with efficiency aspects of water use and not with issues of equity, we suppose that downstream users own the water rights and we abstract from the pricing question. The proper costing of storage facilities and pricing of delivered water is the subject of ongoing research by the authors. We also ignore the role of Federal reserved rights and the possible need for maintaining a minimum "head" for the generation of hydroelectric power (and the potential cost of a deficit), all in an attempt for simplicity. Likewise we recognize but do not formally model the public good properties of stored water: i.e., scenic, recreational, fish and wildlife, ecological, etc. Given these restrictions we can focus on the efficiency aspects of both reservoir manager and downstream user behavior. In particular we show that equal sharing allocations of a waterway are Pareto superior to appropriative allocations (assuming appropriate compensation). Moreover, for given aggregate diversion capacity for firms too much water is stored in the reservoir under appropriation relative to equal sharing. Once the river is completely utilized under appropriation, the introduction of competitive markets in water rights generates an equal sharing allocation: a Pareto optimum is attainable from an appropriative allocation. We also present a theorem on decentralized management for equal sharing: independent optimization by reservoir managers and firms ultimately leads to simultaneously optimal release policies and diversion capacities.

Section II considers a finite lived dam under the equal sharing allocation for stochastic river flows while Section III considers the appropriative analogue. Efficiency results are adduced in Section IV with the decentralization theorem presented in Section V in the analysis of an infinite lived dam. Section VI concludes with some observations on optimal investment in dam size and the relationship between investment in reservoir capacity and investment in diversion capacity.

II. A COMPETITIVE MODEL OF RESERVOIR MANAGEMENT AND WATER USE

A. Optimal Release Policy

We consider the case where a reservoir manager determines an optimal release policy so as to maximize the discounted present value of expected profits accruing to downstream users. The users are assumed to be risk neutral expected profit maximizers all of whom have identical profit functions. The number of firms is N which is fixed; the fixity of N represents an institutional barrier to entry and exit. Unlimited entry can be problematic in that it may lead to incentive incompatibility of the equal sharing doctrine at a long run equilibrium*. For symmetry we preclude exit as well although exit is not troublesome. The presumption of this institutional constraint is not unjustified. For example, the Colorado River Compact (1922) limits use of Colorado River water in both the upper and lower Colorado River Basins to one half of what at that time was thought to be the average annual virgin run-off of the Colorado River. In our framework this amounts to restricting aggregate rights to water to be equal to the expected value of river flows; once N firms have rights which equal expected river flows, entry and exit are precluded. Thus we focus on a short run equilibrium allocation which, in view of the institutional constraint, may become a long run equilibrium under appropriate conditions.

Each of the N firms is assumed to have a profit function, $\pi(d,a)$, where d is deliveries of water received and a is the diversion capacity constructed by each firm. Clearly no firm constructs capacity in excess of his right to receive water;

*See Burness and Quirk [3].

and the limitation of beneficial consumptive use precludes the user from having rights to water in excess of his ability to divert. Thus the firm's diversion capacity can also be viewed as his rights to water. Letting subscripted functions denote partial derivatives, we require that $\pi_1(d,a) \geq 0$, $\pi_2(d,a) < 0$, $\pi_{11}(d,a) < 0$, $\pi_{22}(d,a) \leq 0$. Moreover, we assume that diversion facilities deteriorate only through aging and not by use, so that $\pi_{12}(a,d) \equiv \pi_{12} = 0$. In view of the last assumption, the profit function is additively separable so that, $\pi(d,a) = R(d)-C(a)$, where R and C are revenue and cost functions respectively. Thus $\pi_1(d,a) \equiv \pi_1(d)$ and $\pi_2(d,a) = \pi_2(a)$; we assume that $\pi_2(a) = -c < 0$ for simplicity.

Since a is the diversion capacity of a typical firm, and d, deliveries of water, is constrained by $d \leq a$, the profit funciton $\pi(d,a)$ and the marginal profit ability of water funtion $\pi_1(d,a)$ appear as in Figures 2.1 and 2.2.

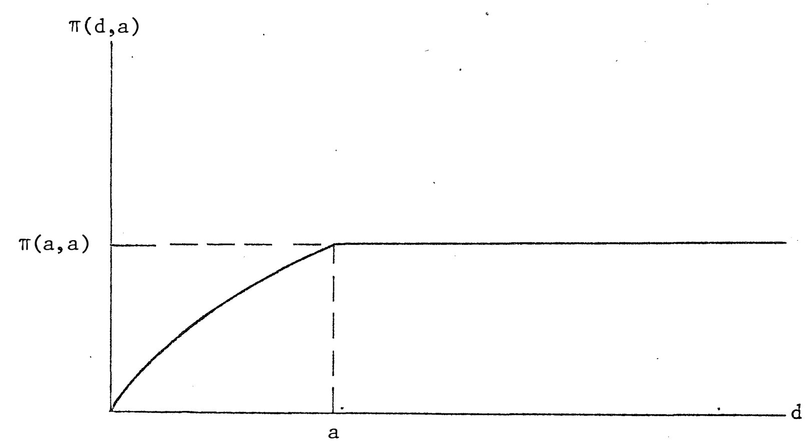

Figure 2.1. The profit function.

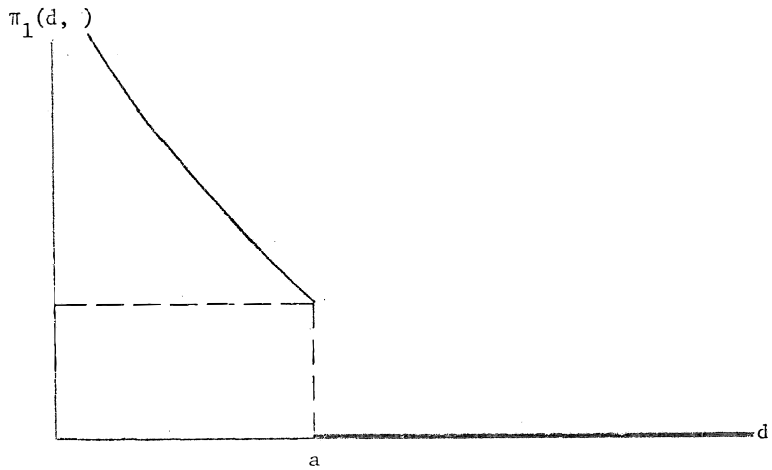

Figure 2.2. The marginal profitability of water.

As indicated in Figures 1 and 2, the profit function has a kink at $d = a$, with marginal profit ability of water positive for $d < a$, and zero for $d > a$. $\pi_1(a,a)$ is not defined. When a is fixed, it is convenient to use the symbol $\pi_1(a,a)$ to refer to the right hand derivative at that point so that $\pi_1(a,a) \equiv 0$.* Note that an implicit assumption in this approach is that there are no flood damages possible; "excess" flows of water simply bypass users of water. Finally, in most of what follows, we will assume that $\pi(d,a)$ satisfies the neoclassical condition that $\lim_{d \to 0} \pi_1(d,a) = +\infty$.

River flows, x_t, are stochastically independent and identically distributed in every time period with probability density function $f(x_t)$. We consider first a T-period planning horizon. As river flows are also inflows into the reservoir, we interpret x_t as inflows when there are t planning periods remaining; y_t represents releases from storage with t periods remaining; k is the rate at which stocks of stored water evaporate $(1-k = \alpha)$; $X_t = \alpha X_{t+1} + x_t - y_t$ is the stock of water with t

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* When a is a variable, that is, when firms choose capacities as well as water usage, then $\pi_1(a,a)$ is treated as $\lim_{d \to a^-} \pi_1(d,a)$.

periods remaining after inflows, releases and evaporation. Inflows are observed by the dam manager, perhaps in the form of upstream precipitation or stream gauge readings, at the beginning of each period after which releases are determined. Thus with t planning periods remaining, water available for use is $R_t = \alpha X_{t+1} + x_t$. The maximum storage capacity of the reservoir is $\bar{X}$.

In the correlative paradigm all firms share equally any release from storage. Consequently, with one period remaining, if releases are y_1, then $d = y_1/N$ for each firm. The reservoir manager wishes to choose y_1 such that $N\pi(y_1/N,a)$ is maximized subject to $0 \le X \le \bar{X}$. Let y_1^* be the smallest value of y_1 such that π is maximized subject to the constraint $d \le a$; since firms cannot use more water than they can divert, clearly $y_1^* = Na$ Thus the optimal one period release policy is

$$(2.1) \qquad \hat{y}_1 = \begin{cases} R_1 & \text{if } R_1 \le Na \\ y_1^* & \text{if } Na \le R_1 \le Na + \bar{X} \\ R_1 - \bar{X} & \text{if } R_1 \ge Na + \bar{X} \end{cases}$$

and aggregate optimal profits with one period remaining are

$$(2.2) \qquad N\pi(\hat{y}_1/N,a) = \begin{cases} N\pi(R_1/N,a) & \text{if } R_1 \le Na \\ N\pi(y_1^*/N,a) & \text{if } R_1 \ge Na \end{cases}$$

Let $\pi^t(y_t, a, R_t)$ represent expected profits for a typical firm when t periods remain, releases are y_t, water available for distribution is R_t and decisions in the remaining t-1 periods are to be made optimally. Define

$$(2.3) \qquad \Pi^t(R_t/N,a) = \max_{0 \le R_t - y_t \le \bar{X}} \pi^t(y_t,a,R_t)$$

with $\Pi^0(\cdot) = 0$. Since $\pi^1(y_1,a,R_1) = \pi(y_1/N,a)$ we have $\Pi^1(R_1/N,a) \equiv \pi(\hat{y}_1/N,a)$

and

$$(2.2')\quad \Pi_1^1(R_1/N,a) = \begin{cases} \pi_1(y_1/N) & \text{if } R_1 < Na \\ 0 & \text{if } R_1 \geq Na \end{cases}$$

so that Π^1 is continuous and concave, Π_1^1 and Π_{11}^1 exist and are continuous* except at y_1*.

With two planning periods remaining the reservoir manager maximizes $N\pi^2(y_2,a,R_2)$ subject to $0 \leq R_2-y_2 \leq \bar{X}$, where

$$(2.4)\quad \pi^2(y_2,a,R_2) = \pi(y_2/N,a) + \beta E\left\{\Pi^1(R_1/N,a)\right\}$$

β being a discount factor $(0<\beta<1)$ and E the expectation operator relative to the distribution $F(x) = \int_0^x f(z)\,dz$.

In view of

$$(2.5)\quad X_t = \alpha X_{t+1} + x_t - y_t$$

and

$$(2.6)\quad R_t = \alpha X_{t+1} + x_t$$

we have

$$(2.7)\quad R_t = \alpha(R_{t+1}-y_{t+1}) + x_t$$

So that (2.4) can be written as

$$(2.8)\quad \pi^2(y_2,a,R_2) = \pi(y_2/N,a) + \beta E\left\{\Pi^1\left[\frac{\alpha(R_2-y_2) + x_1}{N},a\right]\right.$$

Written in full

*Superscripts on functions indicate the number of periods remaining; subscripts on functions denote partial derivatives. Recall that with a fixed, we define $\pi_1(a) \equiv 0$.

$$(2.9)\quad E\left\{\Pi^1\left[\frac{\alpha(R_2-y_2)+x_1}{N},a\right]\right\} = \int_0^{Na-\alpha(R_2-y_2)} \Pi^1\left[\frac{\alpha(R_2-y_2)+x_1}{N},a\right]dF(x)$$

$$+\int_{Na-\alpha(R_2-y_2)}^{Na+\bar{X}-\alpha(R_2-y_2)} \Pi^1\left[\frac{\alpha(R_2-y_2)+x_1}{N},a\right]dF(x) + \int_{Na+\bar{X}-\alpha(R_2-y_2)}^{\infty} \Pi^1\left[\frac{\alpha(R_2-y_2)+x_1}{N},a\right]dF(x)$$

Multiplying (2.8) by N and differentiating the result with respect to y_2 for a given value of R define $\bar{y}_2(R)$ as the smallest value of y_2 such that*

$$(2.10)\quad \pi_1(\bar{y}_2(R)/N) - \alpha\beta E\left\{\Pi_1^1\left[\frac{\alpha(R-\bar{y}_2)+x}{N},a\right]\right\} = 0$$

so that $0 \leq \bar{y}_2(R) \leq Na$. Differentiating (2.10) with respect to R and solving for $\partial\bar{y}_2/\partial R$ yields, for $\bar{y}_2 < Na$,

$$(2.11)\quad \frac{\partial\bar{y}_2}{\partial R} = \frac{\alpha^2\beta E\left\{\Pi_{11}^1\left[\frac{\alpha(R-\bar{y}_2)+x}{N},a\right]\right\}}{\pi_{11}(\bar{y}_s/N,a) + \alpha^2\beta E\left\{\Pi_{11}^1\left[\frac{\alpha(R-\bar{y}_2)+x}{N},a\right]\right\}}$$

So that $0 < \partial\bar{y}_2/\partial R < 1$, for $0 \leq y_2 < Na$. Let $y_2^* = \bar{y}_2(y_2^*)$ be the smallest value of y_2 determined by

$$(2.10')\quad \pi_1(y_2^*/N,a) - \alpha\beta E\left\{\Pi_1^1(x/N,a)\right\} = 0;$$

Given that $\pi(d,a)$ satisfies $\lim_{d\to 0}\pi_1(d) = +\infty$ with $\pi_1(d) > 0$, $\pi_{11}<0$, $d<a$ it follows that $y_2^* > 0$.

*As the subscripts on R and x are now redundant, they are omitted. Note also that $\frac{\partial}{\partial y_2} E\Pi^1 \equiv E\Pi_1^1$.

Let $y_2^{**} = \bar{y}_2(y_2^{**}+\bar{X})$ be the smallest value of y_2 determined by

$$(2.10'')\quad \pi_1(y_2^{**}/N) - \alpha\beta E\left\{\Pi_1^1\left(\frac{\alpha\bar{X}+x}{N},a\right)\right\} = 0$$

Since $\Pi_{11}^1 < 0$, $0 \leq d < Na$, it follows that $y_2^* \leq y_2^{**}$, hence $0 \leq y_2^* \leq y_2^{**} \leq Na$.*

Thus the optimal release policy for the two-period problem is

$$(2.12)\quad \hat{y}_2 = \begin{cases} R & \text{if } R \leq y_2^* \\ \bar{y}_2(R) & \text{if } y_2^* \leq R \leq y_2^{**}+\bar{X} \\ R-\bar{X} & \text{if } R \geq y_2^{**}+\bar{X} \end{cases}$$

and aggregate optimal expected profits are

$$(2.13)\quad N\Pi^2(R/N,a) = \begin{cases} N\pi(R/N,a) + N\beta E\,\Pi^1(x/N,a) & \text{if } R \leq y_2^* \\ N\pi(\bar{y}_2/N,a) + N\beta E\left\{\Pi^1\left[\frac{\alpha(R-\bar{y}_2)+x}{N},a\right]\right\} & \text{if } y_2^* \leq R \leq y_2^{**}+\bar{X} \\ N\pi\left(\frac{R-\bar{X}}{N},a\right) + N\beta E\left\{\Pi^1\left(\frac{\alpha\bar{X}+x}{N},a\right)\right\} & \text{if } R \geq y_2^{**}+\bar{X} \end{cases}$$

while, in view of the fact that $d\hat{y}_2/dR = 1$ on the first and third intervals and (2.10) holds on the second,

$$(2.14)\quad \Pi_1^2(R/N,a) = \begin{cases} \frac{1}{N}\pi_1(R/N) & \text{if } R < y_2^* \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^1\left[\frac{\alpha(R-\bar{y}_2)+x}{N},a\right]\right\} & \text{if } y_2^* \leq R < y_2^{**}+\bar{X} \\ \frac{1}{N}\pi_1\left(\frac{R-\bar{X}}{N}\right) & \text{if } R \geq y_2^{**}+\bar{X} \end{cases}$$

*Further, $0 < y_2^* < Na$ implies $0 < y_2^* < y_2^{**} \leq Na$.

and

$$(2.15)\quad \Pi_{11}^2(R/N,a) = \begin{cases} \frac{1}{N^2}\pi_{11}(R/N) & \text{if } R<y_2^* \\ \frac{\alpha^2\beta E}{N^2}\left\{\Pi_{11}^1\left[\frac{\alpha(R-y_2)+x}{N},a\right]\right\} & \text{if } y_2^* \leq R < y_2^{**}+\bar{X} \\ \frac{1}{N^2}\pi_{11}\left(\frac{R-\bar{X}}{N}\right) & \text{if } R \geq y_2^{**}+\bar{X} \end{cases}$$

so that Π^2 is continuous and concave, Π_1^2 and Π_{11}^2 exist and are continuous, except possibly at y_2^* and $y_2^{**}+\bar{X}$.

With t periods remaining the planner chooses y_t so as to maximize $N\pi^t(y_t,a,R)$ subject to $0 \leq R-y_t \leq \bar{X}$, where

$$(2.16)\quad \pi^t(y_t,a,R) = \pi(x_t/N,a) + \beta E\left\{\Pi^{t-1}\left[\frac{\alpha(R-y_t)+x}{N},a\right]\right\}$$

Similar to the two period problem we can show that optimal releases with t periods remaining are

$$(2.17)\quad \hat{y}_t = \begin{cases} R & \text{if } R \leq y_t^* \\ \bar{y}_t(R) & \text{if } y_t^* \leq R \leq y_t^{**}+\bar{X} \\ R-\bar{X} & \text{if } R \geq y_t^{**}+\bar{X} \end{cases}$$

where y_t^*, $\bar{y}_t(R)$, and y_t^{**} are determined by the t^{th} period analogues to (2.10'), (2.10), and (2.10") respectively. The analogue to (2.11) requires that $0 < \partial\bar{y}_t/\partial R < 1$, for $0 \leq \bar{y}_t \leq Na$. Thus, aggregate optimal expected profits are

$$(2.18)\quad N\Pi^t(R/n,a) = \begin{cases} N\pi(R/N,a) + \beta NE\left\{\Pi^{t-1}(x/N,a)\right\} & \text{if } R \le y_t^* \\ N\pi(\bar{y}_t/N,a) + \beta NE\left\{\Pi^{t-1}\left[\frac{\alpha(R-\bar{y}_t) + x}{N},a\right]\right\} & \text{if } y_t^* \le R \le y_t^{**}+\bar{X} \\ N\pi\left(\frac{R-\bar{X}}{N},a\right) + \beta NE\left\{\Pi^{t-1}\left[\frac{\alpha\bar{X} + x}{N},a\right]\right\} & \text{if } R \ge y_t^{**} + \bar{X} \end{cases}$$

with

$$(2.19)\quad \Pi_1^t(R/N),a) = \begin{cases} \frac{1}{N}\pi_1(R/N) & \text{if } R < y_t^* \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^{t-1}\left[\frac{\alpha(R-\bar{y}_t) + x}{N},a\right]\right\} & \text{if } y_t^* \le R < y_t^{**} + \bar{X} \\ \frac{1}{N}\pi_1\left(\frac{R-\bar{X}}{N}\right) & \text{if } R \ge y_t^{**} + \bar{X} \end{cases}$$

and

$$(2.20)\quad \Pi_{11}^t(R/N,a) = \begin{cases} \frac{1}{N^2}\pi_{11}(R/N) & \text{if } R \le y_t^* \\ \frac{\alpha^2\beta E}{N^2}\left\{\Pi_{11}^{t-1}\left[\frac{\alpha(R-\bar{y}_t) + x}{N},a\right]\right\} & \text{if } y_t^* \le R \le y_t^{**} + \bar{X} \\ \frac{1}{N^2}\pi_{11}\left(\frac{R-\bar{X}}{N}\right) & \text{if } R \ge y_2^{**} + \bar{X} \end{cases}$$

so that, by induction on t, Π^t is continuous and concave, Π_1^t and Π_{11}^t exist and are continuous, except possibly at y_t^* and $y_t^{**} + \bar{X}$.

From equation (2.10") observe that

$$(2.21)\quad \alpha\bar{X} \ge Na$$

implies that $y_2^{**} = Na = y_1^*$ (so that $y_2^* \le y_1^*$); (2.21) implies that the reservoir is large enough so that, if full, its post evaporation contents would suffice to completely satisfy users in the next to the

last and final planning period. If (2.21) were not satisfied, releases equal to Na would never occur with two planning periods remaining: the last units of water would have greater expected marginal profitability if stored for possible use in the following period.

Given (2.21) and recalling (2.14) and the fact that $\Pi^1(R,a) \equiv \hat{\pi}(\hat{y}_1/N,a)$ we have

$$(2.22)\quad \Pi_1^2(R/N,a) - \Pi_1^1(R/N,a) = \begin{cases} 0 & \text{if } R < y_2^* \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^1\left[\frac{\alpha(R-\bar{y}_2) + x}{N},a\right]\right\} - \frac{1}{N}\pi_1(R/N) & \text{if } y_2^* \le R < y_1^* \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^1\left[\frac{\alpha(R-\bar{y}_2) + x}{N},a\right]\right\} & \text{if } y_1^* \le R < y_2^{**} + \bar{X} \\ 0 & \text{if } R \ge y_2^{**} + \bar{X} \end{cases}$$

For $y_2^* < R < y_1^*$ we have $R > \bar{y}_2(R)$, so that $\pi_1(R/N) < \pi_1(\bar{y}_2/N)$ and in view of (2.10), the second line of (2.22) is positive. Hence $\Pi_1^2(R/N,a) - \Pi_1^1(R/N, a) \ge 0$ for all R, with strict inequality for $y_2^* < R < y_1^*$.

We now show that $\Pi_1^t(R/N,a) - \Pi_1^{t-1}(R/N,a) \ge 0$ for all R, with strict inequality on some interval implies $\Pi_1^{t+1}(R/N,a) - \Pi_1^t(R/N,a) \ge 0$ for all R, with strict inequality for some interval. To do this we must assume the generalized form of equation (2.21), namely,

$$(2.23)\quad \bar{X} \ge Na\left(\sum_{s=1}^{t-1}\frac{1}{\alpha^s}\right)$$

This ensures, by the generalized analogue to equation (2.10"), that $y_s^{**} = Na$ for $s = 1, \ldots, t+2$.

Observe that, in view of (2.23),

*Since $\Pi_1^t(R/N,a) > \Pi_1^{t-1}(R/N,a)$ on some intervals, $E\{\Pi_1^t(x/N,a) - \Pi_1^{t-1}(x/N,a)\} > 0$.

$$(2.34)\ \Pi_1^{t+1}(R/N,a) = \begin{cases} \frac{1}{N}\,\pi_1(R/N) & \text{if } R < y^*_{t+1} \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^t\left[\frac{\alpha(R-\bar{y}_{t+1})}{N} + x,a\right]\right\} & \text{if } y^*_{t+1} \le R < y^{**}_{t+1} + \bar{X} \\ 0 & \text{if } R \ge y^{**}_{t+1} + \bar{X} \end{cases}$$

where

$$(2.25)\quad \pi_1(y_t^*/N) - \alpha\beta E\left\{\Pi_1^{t-1}(x/N,a)\right\} = 0$$

and

$$(2.26)\quad \pi_1(y^*_{t+1}/N) - \alpha\beta E\left\{\Pi_1^{t}(x/N,a)\right\} = 0$$

Suppose that $y_t^* \le y^*_{t+1}$. Then $\pi_1(y_t^*/N) \ge \pi_1(y^*_{t+1}/N)$ so that $E\left\{\Pi_1^t(x/N,a) - \Pi_1^{t-1}(x/N,a)\right\} \le 0$ in contradiction to hypothesis. Hence $y_t^* \ge y^*_{t+1}$, with strict inequality if $y^*_{t+1} < Na$. Thus, in view of (2.19) and (2.24)

$$(2.27)\ \Pi_1^{t+1}(R/N,a) - \Pi_1^{t}(R/N,a) = \begin{cases} 0 & \text{if } R < y^*_{t+1} \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^t\left[\frac{\alpha(R-\bar{y}_{t+1})}{N}+x,a\right]\right\} - \frac{1}{N}\pi_1(R/N) & \text{if } y^*_{t+1} \le R < y^*_t \\ \frac{\alpha\beta E}{N}\left\{\Pi_1^t\left[\frac{\alpha(R-\bar{y}_{t+1})}{N}+x,a\right] - \Pi_1^{t-1}\left[\frac{\alpha(R-\bar{y}_{t})}{N}+x,a\right]\right\} & \text{if } y_t^* \le R < Na+\bar{X} \\ 0 & \text{if } R \ge Na + \bar{X} \end{cases}$$

In view of the necessary conditions corresponding to (2.10) and our hypothesis, (2.27) is non-negative with strict inequality for some values of R. In addition we have $\bar{y}_{t+1}(R) > \bar{y}_{t+2}(R)$, otherwise

$E\left\{\Pi_1^{t+1}\left[\frac{\alpha(R-\bar{y}_{t+2})}{N} + x,a\right] - \Pi_1^{t}\left[\frac{\alpha(R-\bar{y}_{t+1})}{N} + x,a\right]\right\} \le 0$, a contradiction.

These results are summarized in Proposition 2.1 and depicted in Figure 2.3.

<u>Proposition 2.1</u>: Suppose that N firms with identical profit functions share equally in the water released from a reservoir with finite capacity and finite life. If the reservoir manager releases water so as to maximize expected profits of water users, then the optimal release policy is given by (2.17). Moreover if $\bar{X} > Na\left(\sum_{s=1}^{t-1}\frac{1}{\alpha^s}\right)$ then $y_j^* \ge y^*_{j+1}$, $y^{**}_{j+1} = Na$ and $\bar{y}_j(R) \ge \bar{y}_{j+1}(R)$ for $j = 1,\ldots,t+1$, with strict inequality if $y^*_{j+1} < Na\ (\bar{y}_{j+1}(R) < Na)$.

B. Optimal Diversion Capacities

In the previous section we deduced the optimal policy for releases from a reservoir, given the diversion capacities of downstream water users. In this section we analyze the response of firms to an announced policy for releases. The firm must, in view of the announced policy, make a once and for all decision concerning the diversion capacity (rights to water) which they desire. For the time being we maintain the fiction that the number of firms is fixed, but relax, if necessary, the assumption that the aggregate claims to water cannot exceed the expected value of stream flows. More will be said on this in sections V and VI. For simplicity we maintain the assumption that $\bar{X} \ge Na\left(\sum_{s=1}^{t-1}\frac{1}{\alpha^s}\right)$ so that $y^{**}_{j+1} = Na$ for $j = 1, \ldots, t$. We also assume that no firm takes into account the effect of its choice of a capacity on the policies adopted by the dam manager.

Expected profits for a representative firm, with t periods remaining until the end of the horizon, are, from equation (2.18).

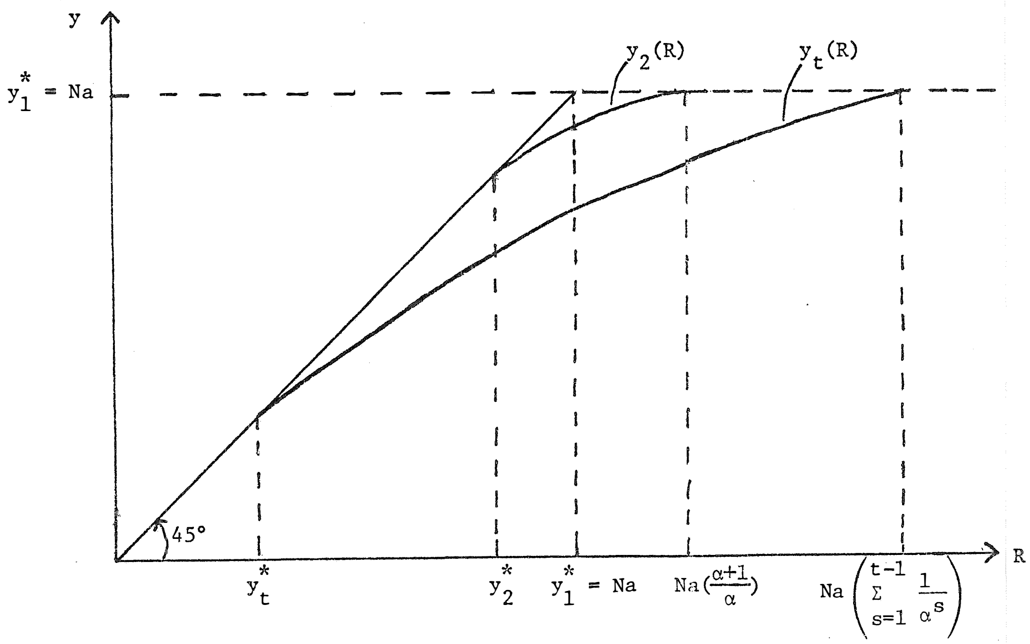


Figure 2.3: Optimal Releases as a Function of Water Available: Finite Capacity and Finite Lived Reservoir.

$$(2.28)\quad \Pi^t(R/N,a) = \begin{cases} \pi(R/N,a) + \beta E\,\{\Pi^{t-1}(x/N,a) & \text{if } R \le y_t^* \\ \pi(\bar{y}_t/N,a) + \beta E\,\{\Pi^{t-1}[\frac{\alpha(R-\bar{y}_t)+x}{N},a]\} & \text{if } y_t^* \le R \le y_t^{**}+\bar{X} \\ \pi(\frac{R-\bar{X}}{N},a) + \beta E\,\{\Pi^{t-1}(\frac{\alpha\bar{X}+x}{N},a)\} & \text{if } R \ge y_t^{**}+\bar{X} \end{cases}$$

Optimal diversion capacity is determined by differentiating (2.28) with respect to a and setting the result equal to zero. Performing the differentiation yields, recalling that $\pi_2(a) = -c < 0$,

$$(2.29)\quad \Pi_2^t(R/N,a) = \begin{cases} -c + \beta\,\dfrac{dE\{\Pi^{t-1}(x/N,a)\}}{da} & \text{if } R \le y_t^* \\ -c + \beta\,\dfrac{dE\{\Pi^{t-1}[\frac{\alpha(R-\bar{y}_t)+x}{N},a]\}}{da} & \text{if } y_t^* \le R \le y_t^{**} + \bar{X} \\ -c + \beta\,\dfrac{dE\{\,^{t-1}\left(\frac{\alpha\bar{X}+x}{N},\ a\right)\}}{da} & \text{if } R \ge y_t^{**} + \bar{X} \end{cases}$$

which, upon expanding the expectations is,*

---

*When the level of diversion capacity a is a variable, then a change in a also changes the amount of water diverted d, when the capacity constraint is effective. Hence $\Pi_2^t(R/N,a)$ is to be interpreted as the change in Π^t when a is changed, given that d changes together with a when the capacity constraint is effective. Moreover, when a is variable, $\pi_1(a)$, and $\Pi_1^t(a,a)$ exist and generally are non-zero.

$$
(2.29')\quad \Pi_2^t(R,a)=\begin{cases}
-c+\beta\left\{\displaystyle\int_0^{y^*_{t-1}}\Pi_2^{t-1}(x/N,a)\,dF(x)+\int_{y^*_{t-1}}^{y^{**}_{t-1}+\bar{X}}\Pi_2^{t-1}\left(\frac{\bar{y}_{t-1}}{N},a\right)dF(x)+\right. \\
\qquad\left.\displaystyle\int_{y^{**}_{t-1}+\bar{X}}^{\infty}[\Pi_2^{t-1}(a,a)+\Pi_1^{t-1}(a,a)]\,dF(x)\right\} & \text{if } R\le y_t^* \\
-c+\beta\left\{\displaystyle\int_0^{y^*_{t-1}-\alpha(R-\bar{y}_t)}\Pi_2^{t-1}\left[\frac{\alpha(R-\bar{y}_t)+x}{N},a\right]dF(x)+\int_{y^*_{t-1}-\alpha(R-\bar{y}_t)}^{y^{**}_{t-1}+\bar{X}-\alpha(R-\bar{y}_t)}\Pi_2^{t-1}\left(\frac{\bar{y}_{t-1}}{N},a\right)dF(x)+\right. \\
\qquad\left.\displaystyle\int_{y^{**}_{t-1}+\bar{X}-\alpha(R-\bar{y}_t)}^{\infty}[\Pi_2^{t-1}(a,a)+\Pi_1^{t-1}(a,a)]\,dF(x)\right\} & \text{if } y_t^*\le R\le y_t^{**}+\bar{X} \\
-c+\beta\left\{\displaystyle\int_0^{y^*_{t-1}-\alpha\bar{X}}\Pi_2^{t-1}\left(\frac{\alpha\bar{X}+x}{N},a\right)dF(x)+\int_{y^*_{t-1}-\alpha\bar{X}}^{y^{**}_{t-1}+(1-\alpha)\bar{X}}\Pi_2^{t-1}\left(\frac{\bar{y}_{t-1}}{N},a\right)dF(x)+\right. \\
\qquad\left.\displaystyle\int_{y^{**}_{t-1}+(1-\alpha)\bar{X}}^{\infty}[\Pi_2^{t-1}(a,a)+\Pi_1^{t-1}(a,a)]\,dF(x)\right\} & \text{if } R\ge y_t^{**}+\bar{X}
\end{cases}
$$

Recalling that $\Pi^o \equiv 0$ and $y_1^* = Na$, $\Pi^1(R/N,a) \equiv \pi(\hat{y}_1/N,a)$ and $\pi_{12} = 0$ imply that $\Pi_{12}^1 = 0$ so that for $t = 2$,

$$
(2.30)\ \Pi_2^2(R/N,a) = \begin{cases}
-c+\beta\left\{\Pi_2^1+[1-F(Na)]\Pi_1^1(a,a)\right\} & \text{if } R\le y_2^* \\
-c+\beta\left\{\Pi_2^1+[1-F(Na-\alpha(R-\bar{y}_2))]\Pi_1^1(a,a)\right\} & \text{if } y_2^*\le R\le y_2^{**}+\bar{X} \\
-c+\beta\left\{\Pi_2^1+[1-F(Na-\alpha\bar{X})]\Pi_1^1(a,a)\right\} & \text{if } R\ge y_2^{**}+\bar{X}
\end{cases}
$$

where $\Pi_2^1 \equiv \pi_2(a)$ and $\Pi_1^1(a,a) = \pi_1(a)$. From (2.30) it is clear that

$$
(2.31)\ \Pi_{12}^2(R/N,a) = \begin{cases}
0 & \text{if } R\le y_2^* \\
\alpha\beta f[Na-\alpha(R-\bar{y}_2)]\Pi_1^1(a,a)\left(1-\frac{\partial\bar{y}_2}{\partial R}\right) & \text{if } y_2^*\le R\le y_2^{**}+\bar{X} \\
0 & \text{if } R\ge y_2^{**}+\bar{X}
\end{cases}
$$

and

$$
(2.32)\ \Pi_{22}^2(R/N,a) = \begin{cases}
\beta\left\{[1-F(Na)]\Pi_{11}^1(a,a)-Nf(\cdot)\Pi_1^1(a,a)+\Pi_{22}^1\right\} & \text{if } R\le y_2^* \\
\beta\left\{[1-F(Na-\alpha(R-\bar{y}_2))]\Pi_{11}^1(a,a)-Nf(\cdot)\Pi_{11}^1(a,a)+\Pi_{22}^1\right\} & \text{if } y_2^*\le R\le y_2^{**}+\bar{X} \\
\beta\left\{[1-F(Na-\alpha\bar{X})]\Pi_{11}^1(a,a)-Nf(\cdot)\Pi_1^1(a,a)\right\}+\Pi_{22}^1\} & \text{if } R\ge y_2^{**}+\bar{X}
\end{cases}
$$

so that $\Pi_{12}^2 \ge 0$ and $\Pi_{22}^2 < 0$. From (2.29') and (2.31) one can easily arrive at the conclusion that $\Pi_{12}^t > 0$ for $t = 3,\ldots,T$ while $\Pi_{22}^t < 0$ for $t = 1,\ldots,T$.

In the event that no dam exists equation (2.28) becomes*

$$(2.33)\ \Pi^t(x/N,a) = \pi(x/N,a) + \beta E\{\Pi^{t-1}(x/N,a)\}$$

and (2.29') becomes

*Recall that x in $\pi(x/N,a)$ is observed at the beginning of period t and hence $\Pi^t(x/N,a)$ is deterministic.

$$(2.34)\quad \Pi_2^t(x/N,a) = -c+\beta\left\{\int_0^\infty \Pi_2^{t-1}(x/N,a)dF(x) + \int_{Na}^\infty \Pi_1^{t-1}(x/N,a)dF(x)\right\}$$

so that $\Pi_{12}^t(x/N,a) = 0$ and $\Pi_{22}^t(x/N,a) < 0$ for all t. Since $R > x$ for $\bar{X} > 0$ we have

$$(2.35)\quad \Pi_2^t(R/N,a) > \Pi_2^t(x/N,a).$$

Equation (2.35) leads directly to

Proposition 2.2: Under equal sharing of water among N identical expected profit maximizing firms with separable profit functions, water users find it optimal to construct greater diversion capacity when a reservoir is operated optimally, in the sense of Proposition 2.1, than when no dam exists at all (i.e., when no water is ever stored in the reservoir).

III. APPROPRIATIVE WATER RIGHTS

In this section we parallel the development of the previous section for the case of appropriative water rights. The one period profit function is $g(y,a) \equiv \pi(y,a)$, but for the most part, similarities end there. In particular, the rule for determining optimal releases is not straightforward as a result of non-convexities in the aggregate profit function.

A. Optimal Releases

Under the appropriative doctrine prior appropriators have seniority in rights to those rights holders appropriating later in time. We label appropriators in order of decreasing priority; i.e., the first appropriator has rights in the amount a_1 and his claims to water must be completely satisfied before other appropriators receive water. Similarly, if a_i is the i^{th} appropriator's right to water, then $A_{i-1} = \sum_{j=1}^{i-1} a_j$ represents the claims to water senior to those of the i^{th} appropriator; the i^{th} appropriator receives no water unless releases exceed A_{i-1}. Consequently, with one period remaining, profits for the i^{th} appropriator are

$$(3.1)\quad g^1(y_1,a_i,R) = \begin{cases} g(o,a_i) & \text{if } y_1 \leq A_{i-1} \\ g(y_1-A_{i-1},a_i) & \text{if } A_{i-1} \leq y_1 \leq A_i \\ g(a_i,a_i) & \text{if } y_1 \geq A_i \end{cases}$$

$$(A_\circ \equiv 0)$$

and*

$$(3.2)\quad \frac{\partial g^1}{\partial y_1} = \begin{cases} 0 & \text{if } y_1 < A_{i-1} \\ g_1(y_1 - A_i) & \text{if } A_{i-1} \leq y < A_i \\ 0 & \text{if } y_1 \geq A_i \end{cases}$$

so that the smallest value of y_1 which maximizes $\sum_{i=1}^{N} g^1(y_1, a_i, R), y_1^*$, satisfies

$y_1^* = A_N$. Thus optimal one period release policy is

$$(3.3)\quad \hat{y}_1 = \begin{cases} R & \text{if } R \leq A_N \\ A_N & \text{if } A_N \leq R \leq A_N + \bar{X} \\ R - \bar{X} & \text{if } R \geq A_N + \bar{X} \end{cases}$$

Let $G^1(R-A_{i-1}, a_i) = \max\limits_{0 \leq R - y_1 \leq \bar{X}} g^1(y_1, a_i, R) \equiv g^1(\hat{y}_1, a_i R)$. Then

$$(3.4)\quad G^1(R-A_{i-1}, a_i) = \begin{cases} g(o, a_i) & \text{if } R \leq A_{i-1} \\ g(R-A_{i-1}, a_i) & \text{if } A_{i-1} \leq R \leq A_i \\ g(a_i, a_i) & \text{if } R \geq A_i \end{cases}$$

with

$$(3.5)\quad G_1^1(R-A_{i-1}, a_i) = \begin{cases} 0 & \text{if } R < A_{i-1} \\ g_1(R-A_{i-1}) & \text{if } A_{i-1} \leq R < A_i \\ 0 & \text{if } R \geq A_i \end{cases}$$

Thus aggregate optimal profits with one period remaining are

---

*Since $g(y,a) \equiv \pi(y,a)$, $\pi_{12} = 0$ implies that $g_1(y,a) \equiv g_1(y)$.

$$(3.6)\quad \sum_{i=1}^{N} G^1(R-A_{i-1}, a_i) = \begin{cases} g(R, a_1) + \sum_{i=2}^{N} g(o, a_i) & \text{if } R \leq A_1 \\ \vdots & \\ \sum_{i=1}^{j-1} g(a_i, a_i) + g(R-A_{j-1}, a_j) + \sum_{i=j+1}^{N} g(o, a_i) & \\ \vdots & \text{if } A_{j-1} \leq R \leq A_j \\ \sum_{i=1}^{N} g(a_i, a_i) & \text{if } R \geq A_N \end{cases}$$

and

$$(3.7)\quad \sum_{i=1}^{N} G_1^1(R-A_{i-1}, a_i) = \begin{cases} g_1(R) & \text{if } R < A_1 \\ \vdots & \\ g_1(R-A_{j-1}) & \text{if } A_j \leq R < A_j \\ \vdots & \\ 0 & \text{if } R \geq A_N \end{cases}$$

Note that $G_{11}^1 \leq 0$ where G_{11}^1 exists but that G_1^1 and G_{11}^1 do not exist at $A_1, A_2, \ldots A_N$.

With two planning periods remaining expected profits for the i^{th} firm are

$$(3.8)\quad g^2(y_2, a_i, R) = \begin{cases} g(o, a_i) + \beta \left\{ \int_0^{A_{i-1} - \alpha(R-y_2)} G^1(o, a_i)\, dF(x) + \int_{A_{i-1} - \alpha(R-y_2)}^{A_i - \alpha(R-y_2)} G^1[\alpha(R-y_2) - A_{i-1} + x, a_i]\, dF(x) + \int_{A_i - \alpha(R-y_2)}^{\infty} G^1(a_i, a_i)\, dF(x) \right\} & \text{if } y_2 \leq A_{i-1} \\ g(y_2 - A_{i-1}, a_i) + \beta\{\cdot\} & \text{if } A_{i-1} \leq y_2 \leq A_i \\ g(a_i, a_i) + \beta\{\cdot\} & \text{if } y_2 \geq A_i \end{cases}$$

$$= \begin{cases} g(o,a_i) + \beta\Big\{ F[A_{i-1}-\alpha(R_2-y_2)]\, G^1(o,a_i) + \displaystyle\int_{A_{i-1}\alpha(R-y_2)}^{A_i-\alpha(R-y_2)} G^1[\alpha(R-y_2) - A_{i-1}+x, a_i]\,dF(x) \\ \qquad + [1-F(A_i-\alpha(R_2-y_2))]G^1(a_i,a_i)\Big\} & \text{if } y_2 \le A_{i-1} \\ g(y_2-A_{i-1},a_i) + \beta\{\cdot\} & \text{if } A_{i-1} \le y_2 \le A_i \\ g(a_i,a_i) + \beta\{\cdot\} & \text{if } y_2 \ge A_i \end{cases}$$

so that

$$(3.9)\quad \sum_{i=1}^{N} g^2(y_2,a_i,R) = \begin{cases} g(y_2,a_1) + \displaystyle\sum_{i=2}^{N} g(o,a_i) + \beta\Big\{\sum_{i=1}^{N} F(A_{i-1}-\alpha(R-y_2))G^1(o,a_i) \\ \quad + \displaystyle\sum_{i=1}^{N} \int_{A_{i-1}-\alpha(R-y_2)}^{A_i-\alpha(R-y_2)} G^1(\alpha(R_2-y_2) - A_{i-1}+x, a_i)\,dF(x) \\ \quad + \displaystyle\sum_{i=1}^{N} [1-F(A_i-\alpha(R-y_2))]G^1(a_i,a_i)\Big\} & \text{if } y_2 \le A_1 \\ \quad \vdots & \\ \displaystyle\sum_{i=1}^{j-1} g(a_i,a_i) + g(y_2-A_{j-1},a_j) + \sum_{i=j+1}^{N} g(o,a_i) + \beta\{\cdot\} & \\ \quad \vdots & \\ & \text{if } A_{j-1} \le y_2 \le A_j \\ \displaystyle\sum_{i=1}^{N-1} g(a_i,a_i) + g(y_2-A_{N-1},A_N) + \beta\{\cdot\} & \text{if } A_{N-1} \le y_2 \le A_N \\ \displaystyle\sum_{i=1}^{N} g(a_i,a_i) + \beta\{\cdot\} & \text{if } y_2 \ge A_N \end{cases}$$

and

$$(3.10)\quad \sum_{i=1}^{N} \frac{\partial g^2}{\partial y_2} = \begin{cases} g_1(y_2) \quad - \alpha\beta \displaystyle\sum_{i=1}^{N} \int_{A_{i-1}-\alpha(R-y_2)}^{A_i-\alpha(R-y_2)} G_1^1[\alpha(R-y_2) - A_{i-1}+x, a_i]\,dF(x) & \\ \quad \vdots & \text{if } y_2 < A_1 \\ g_1(y_2-A_{j-1}) \quad - \alpha\beta \displaystyle\sum_{i=1}^{N} \int_{A_{i-1}-\alpha(R-y_2)}^{A_i-\alpha(R-y_2)} G_1^1[\alpha(R-y_2)-A_{i-1}+x, a_i]\,dF(x) & \\ \quad \vdots & \text{if } A_{j-1} \le y_2 < A_j \\ -\alpha\beta \displaystyle\sum_{i=1}^{N} \int_{A_{i-1}-\alpha(R-y_2)}^{A_i-\alpha(R-y_2)} G_1^1[\alpha(R-y_2)-A_{i-1}+x, a_i]\,dF(x) & \text{if } y_2 \ge A_N \end{cases}$$

Define $y_2{}^*(j)$ as the smallest value of y_2, $A_{j-1} \le y_2 \le A_j$ satisfying

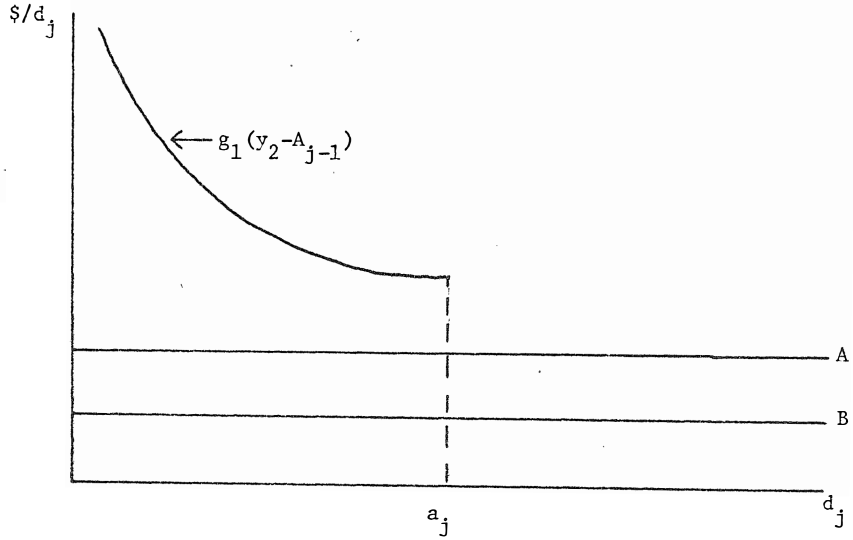
$$(3.11)\quad 0 = g_1(y_2{}^*(j)-A_{j-1}) \quad - \alpha\beta \sum_{i=1}^{N} \int_{A_{i-1}}^{A_i} G_1^1(x-A_{i-1},a_i)\,dF(x),\ j = 1, \ldots, N.$$

$y_2^*(j)$ can be interpreted as the smallest value of $R \ge A_{j-1}$ such that if $R = y_2^*(j)$ then $\sum_{i=1}^{N} g^2$ is maximized by choosing $y_2 = y_2^*(j)(=R)$. Clearly $y^*{}_2(j) > A_{j-1}$, assuming $\lim_{d\to 0} g(d,a) = +\infty$. Similarly define $y_2^{**}(j)$ as the smallest value of y_2, $A_{j-1} \le y_2 \le A_j$ which satisfies

$$(3.12)\quad 0 = g_1(y_2^{**}(j)-A_{j-1}) \quad -\alpha\beta \sum_{i=1}^{N} \int_{A_{i-1}-\alpha\bar{X}}^{A_i-\alpha\bar{X}} G_1^1(\alpha\bar{X}+x-A_{i-1},a_i)\,dF(x)$$

for $j=1,\ldots,N$. $y_2^{**}(j)$ can be interpreted as the smallest value of $R-\bar{X} \ge A_{j-1}$ such that if $R-\bar{X} = y_2^{**}(j)$, then $\sum_{i=1}^{N} g^2$ is maximized by choosing $y_2 = y_2^{**}(j)$.

It should be noted that because of the kink in g(d,aj) at g(aj,aj), it might be the case that no values $y_2^*(j)$, $y_2^{**}(j)$ exist satisfying (3.11) and (3.12). The case is as depicted below



where $A = \alpha\beta \sum_{i=1}^{N} \int_{A_{i-1}}^{A_i} G_1^1(x - A_{i-1}, a_i)\,dF(x)$,

$$B = \alpha\beta \sum_{i=1}^{N} \int_{A_{i-1}-\alpha\bar{X}}^{A_i-\alpha\bar{X}} G_1^1(\alpha\bar{X} + x - A_{i-1}, a_i)\,dF(x).$$

In the case shown in the figure, (3.10) and (3.11) are not satisfied. As a matter of convention, we define $y_2^*(j) = y_2^{**}(j) = A_j$ for this case. Because $\lim_{d\to 0} g_1(d) = +\infty$, then, so long as A and B satisfy $A > g_1(a_j)$, $B > g_1(a_j)$, (3.10) and (3.11) will always be satisfied with $y_2^*(j) < A_j$, $y_2^{**}(j) < A_j$.

We also note that in the case $A > g_1(a_j)$, $B > g_1(a_j)$, $A_{j-1} < y_2^*(j) < y_2^{**}(j) < A_j$. To see this, consider

$$\frac{d}{d(\alpha\bar{X})} \sum_{i=1}^{N} \int_{A_{i-1}-\alpha\bar{X}}^{A_i-\alpha\bar{X}} G_1^1(\alpha\bar{x} + x - A_{i-1}, a_i)\,dF(x) =$$

$$G_1^1(o, a_i) f(-\alpha\bar{X}) - G_1^1(a_N, a_N) f(A_N - \alpha\bar{X})$$

$$+ \sum_{i=1}^{N} \int_{A_{i-1}-\alpha\bar{X}}^{A_i - \bar{X}} G_{11}^1(\alpha\bar{X} + x - A_{i-1}, a_i)\,dF(x) < 0 \text{ for } \alpha\bar{X} \geq 0$$

since $f(x) = 0$ for $x < 0$, $\lim_{x\to 0} G_1^1(x, a_i) f(x) = 0$ by convergence of the integral $G_1^1(x, a_i) \geq 0$, $G_{11}^1(x, a_i) < 0$.

Hence, from (3.10) and (3.11) we have

$$A_{j-1} < y_2^*(j) \leq y_2^{**}(j) \leq A_j$$

with strict inequalities if $A > g_1(a_j)$, $B > g_1(a_j)$. Generally $\bar{y}_2(R,j)$ satisfies

$$0 = g_1[\bar{y}_2(R,j) - A_{j-1}] - \alpha\beta \sum_{i=1}^{N} \int_{A_{i-1}-\alpha(R-\bar{y}_2)}^{A_i - \alpha(R-\bar{y}_2)} G_1^1[\alpha(R - \bar{y}_2) - A_{i-1} + x, a_i]\,dF(x),$$

when such a $\bar{y}_2$ exists ($\bar{y}_2(R,j) = A_j$ otherwise), with $y_2^*(j) \leq R \leq y_2^{**}(j) + \bar{X}$. Thus with two periods remaining the policy for optimal releases is

$$
(3.14)\quad \hat{y}_2 = \begin{cases}
R & \text{if } R \leq y_2^*(1) \\
\bar{y}_2(R,1) & \text{if } y_2^*(1) \leq R \leq y_2^{**}(1) + \bar{X} \\
R-\bar{X} & \text{if } y_2^{**}(1) + \bar{X} \leq R \leq \bar{X} + A_1 \\
R & \text{if } \bar{X} + A_1 \leq R \leq y_2^*(2) \\
\bar{y}_2(R,2) & \text{if } y_2^*(2) \leq R \leq y_2^{**}(2) + \bar{X} \\
R-\bar{X} & \text{if } y_2^{**}(2) + \bar{X} \leq R \leq \bar{X} + A_2 \\
 & \vdots \\
R & \text{if } \bar{X} + A_{j-1} \leq R \leq y_2^*(j) \\
\bar{y}_2(R,j) & \text{if } y_2^*(j) \leq R \leq y_2^{**}(j) + \bar{X} \\
R-\bar{X} & \text{if } y_2^{**}(j) + \bar{X} \leq R \leq \bar{X} + A_j \\
 & \vdots \\
R & \text{if } \bar{X} + A_{N-1} \leq R \leq y_2^*(N) \\
\bar{y}_2(R,N) & \text{if } y_2^*(N) \leq R \leq y_2^{**}(N) + \bar{X} \\
R-\bar{X} & \text{if } R \geq y_2^{**}(N) + \bar{X}
\end{cases}
$$

From (3.14) the distortive effect of the appropriative doctrine is obvious; water is released to the first appropriator until at the margin the expected aggregate present value of future use of stored water exceeds the current marginal profitability of water use by the first appropriator. As the appropriative doctrine prohibits the second appropriator from receiving water until the first appropriator's claims are completely satisfied, water is stored until either the reservoir is full and spilled water satisfies the first appropriator or the present marginal expected profitability of future water use falls below

the current marginal expected profitability of water use by the first appropriator.* Only then do deliveries to the second appropriator commence. A similar procedure is followed for the second appropriator, etc.

Intuitively what is happening is that in the early years after a dam is built, water beyond the needs of the senior appropriator tends to be stored. As capacity of the dam is reached, the river begins to look much like a certain stream flow for senior appropriators and like an undammed river with average flows of less than the expected value of stream flows to junior appropriators.

For convenience we write optimal expected profits for the i^{th} appropriator as

$$
(3.15)\quad G^2(R-A_{i-1},a_i) = \begin{cases}
g(o,a_i)+\beta\Big\{F[A_{i-1}-\alpha(R-\hat{y}_2)]G^1(o,a_i) \\
\quad + \displaystyle\int_{A_{i-1}-\alpha(R-\hat{y}_2)}^{A_i-\alpha(R-\hat{y}_2)} G^1[\alpha(R-\hat{y}_2) + x - A_{i-1},a_i]dF(x) \\
\quad + [1-F(A_i-\alpha(R-\hat{y}_2))]G^1(a_i,a_i)\Big\} & \text{if } R \leq \bar{X} + A_{i-1}. \\
g(R-A_{i-1},a_i) + \beta\{\cdot\} & \text{if } \bar{X}+A_{i-1} \leq R \leq y_2^*(i) \\
g[\bar{y}_2(R,i)-A_{i-1},a_i]+\beta\{\cdot\} & \text{if } y_2^*(i) \leq R \leq y_2^{**}(i)+\bar{X} \\
g(R-\bar{X}-A_{i-1},a_i)+\beta\{\cdot\} & \text{if } y_2^{**}(i)+\bar{X} \leq R \leq \bar{X}+A_i \\
g(a_i,a_i)+\beta\{\cdot\} & \text{if } R \geq \bar{X}+A_i
\end{cases}
$$

---

*If $y_2^{**}(1) = A_1$, then the solution is greatly simplified. Observe that if $y_2^{**}(j) = A_j$, then $g_{11} < 0$ and $a_j > a_i$ for $j < i$ imply $y_2^{**}(i) = A_i$ for $i > j$. We later establish $a_j > a_i$ for $j < i$. In the case $y_2^{**}(1) = A_1$, deliveries to the second appropriator occur when $R = \bar{X} + A_1$, to the third when $R = \bar{X} + A_2$, etc.

since writing out the explicit form for (3.15) would require the substitution of $\hat{y}_2$ from (3.14) into the discounted term on the right hand side of (3.15). In doing this one would obtain $3(i-1)$ terms replacing the first line of (3.15) and $3(N-i)$ terms replacing the last line; i.e., written out completely (3.15) involves 3N expressions, one holding on each of 3N intervals.

In a manner similar to that above we can show that there exist $y_t^*(j)$, $y_t^{**}(j)$, $\bar{y}_t(R,j)$, (subject to the same qualifications holding at $A_1, \ldots A_N$ as noted above) $j = 1, \ldots, N$, such that the optimal release policy with t periods remaining is

$$(3.16)\quad \hat{y}_t = \begin{cases} R & \text{if } R \le y_t^*(1) \\ \bar{y}_t(R,1) & \text{if } y_t^*(1) \le R \le y_t^{**}(1) + \bar{X} \\ R-\bar{X} & \text{if } y_t^{**}(1) + \bar{X} \le R \le \bar{X} + A_1 \\ \vdots & \\ R & \text{if } \bar{X} + A_{j-1} \le R \le y_t^*(j) \\ \bar{y}_t(R,j) & \text{if } y_t^*(j) \le R \le y_t^{**}(j) + \bar{X} \\ R-\bar{X} & \text{if } y_t^{**}(j) + \bar{X} < R < \bar{X} + A_j \\ \vdots & \\ R & \text{if } \bar{X} + A_{N-1} \le R \le y_N^*(j) \\ \bar{y}_t(R,N) & \text{if } y_t^*(N) \le R \le y_t^{**}(N) + \bar{X} \\ R-\bar{X} & \text{if } R \ge y_t^{**}(N) + \bar{X} \end{cases}$$

and, in most compact form, expected profits for the i^{th} appropriator are

$$(3.17)\quad G^t(R-A_{i-1},a_i) = \begin{cases} g(o,a_i)+\beta E\{G^{t-1}[\alpha(R-\hat{y}_t)+x-A_{i-1},a_i]\} & \\ \qquad\qquad\qquad\qquad \text{if } R \le \bar{X} + A_{i-1} & \\ g(R-A_{i-1},a_i)+\beta E\{G^{t-1}(\cdot)\} & \text{if } \bar{X}+A_{i-1} \le R \le y_t^*(i) \\ g[\bar{y}_t(R,i),a_i]+\beta E\{G^{t-1}(\cdot)\} & \text{if } y_t^*(i) \le R \le y_t^{**}(i)+\bar{X} \\ g(R-X-A_{i-1},a_i)+\beta E\{G^{t-1}(\cdot)\} & \text{if } y_t^{**}(i)+\bar{X} \le R \le \bar{X}+A_i \\ g(a_i,a_i) + \beta E\{G^{t-1}(\cdot)\beta\} & \text{if } R \ge \bar{X} + A_i \end{cases}$$

Proposition 3.1: Under the appropriative doctrine of water rights, a reservoir manager who maximizes the aggregate expected profitability of downstream rights owners follows the pattern of optimal releases given by (3.16).

Remark: As the aggregate profit function is not concave under the appropriative doctrine, the ordering of releases, as in Proposition 2.1, cannot be shown.

B. Optimal Diversion Capacity

Given that releases are as in (3.16) consider the choices of diversion capacities by firms facing this set of release rules. Differentiating (3.17) with respect to a_i yields

$$(3.18)\quad \frac{dG^t}{da_i}(R-A_{i-1},a_i) = \begin{cases} g_2 + \beta \frac{d}{da_i} E\{G^{t-1}[\alpha(R-y_t) + x - A_{i-1},a_i]\} & \text{if } R \le X + A_{i-1} \\ g_2 + \beta \frac{d}{da_i} E\{G^{t-1}(\cdot)\} & \text{if } \bar{X} + A_{i-1} \le R \le y_t^*(i) \\ g_2 + \beta \frac{d}{da_i} E\{G^{t-1}(\cdot)\} & \text{if } y_t^*(i) \le R \le y_t^{**}(i) + \bar{X} \\ g_2 + \beta \frac{d}{da_i} E\{G^{t-1}(\cdot)\} & \text{if } y_t^{**}(i) + \bar{X} \le R \le \bar{X} + A_i \\ g_2 + \beta \frac{d}{da_i} E\{G^{t-1}(\cdot)\} & \text{if } R \ge X + A_i \end{cases}$$

and, letting $\hat{y}_t' = \partial\hat{y}_t/\partial R$ while recalling that $g_{21} = 0$

$$(3.19)\quad G_{21}^{t}(R-A_{i-1},a_i) = \begin{cases} (1-\hat{y}_t')\alpha\beta E\{G_{21}^{t-1}[\alpha(R-\hat{y}_2)+x-A_{i-1},a_i]\} & \text{if } R \geq \bar{X}+A_{i-1} \\ (1-\hat{y}_t')\alpha\beta E\{G_{21}^{t-1}(\cdot)\} & \text{if } \bar{X}+A_{i-1} \leq R \leq y_t^*(i) \\ (1-\hat{y}_t')\alpha\beta E\{G_{21}^{t-1}(\cdot)\} & \text{if } y_t^*(i) \leq R \leq y_t^{**}(i)+\bar{X} \\ (1-\hat{y}_t')\alpha\beta E\{G_{21}^{t-1}(\cdot)\} & \text{if } y_t^{**}(i)+\bar{X} \leq R \leq \bar{X}+A_i \\ (1-\hat{y}_t')\alpha\beta E\{G_{21}^{t-1}(\cdot)\} & \text{if } R \geq \bar{X}+A_i \end{cases}$$

In (3.19) $\hat{y}_t$ takes on different values dependent on the value of R as defined by (3.16). In particular $\hat{y}_t' = 1$ on the first and third intervals of each triplet in (3.16), while on the second interval of each triplet $\bar{y}_t(R,j)$ is defined by the necessary condition

$$(3.20)\quad 0 = g_1[\bar{y}_t(R,j)-A_{j-1}] - \alpha\beta\sum_{i=1}^{N} E\{G_1^{t-1}[\alpha(R-\bar{y}_t(R,j))+x-A_{i-1},a_i]\}$$

for $j = 1,\ldots,N$. Differentiating (3.20) with respect to R and solving for $\partial\bar{y}_t(R,j)/\partial R \equiv \bar{y}_t'(R,j)$ yields, for $j = 1,\ldots,N$,

$$(3.21)\quad \bar{y}_t'(R,j) = \frac{\alpha\beta\sum_{i=1}^{N} E\{G_{11}^{t-1}[\alpha(R-\bar{y}_t)+x-A_{j-1},a_j]\}}{g_{11}(\bar{y}_t-R)+\alpha\beta\sum_{i=1}^{N} E\{G_{11}^{t-1}[\alpha(R-\bar{y}_t)+x-A_{j-1},a_j]\}}$$

By an induction argument $E\{G_{11}^{t-1}(\cdot)\} < 0$ for all t so that $y_t^*(j) < y_t^{**}(j)$ implies $0 < \bar{y}_t'(R,j) < 1$ for $t = 1,\ldots,T$ and $j = 1,\ldots,N$. With $0 < y_t' < 1$, another induction argument yields $G_{21}^{t}(R-A_{i-1},a_i) \geq 0$ in (3.19).

Precisely what the implications of these conditions are for distribution of diversion capacities is problematic. If we consider the contrast between a situation in which there is no dam ($\bar{X} = 0$) and one in which there is a dam operating to maximize expected profits according to the appropriative rights scheme, the following intuitive comments might be made.

First, the capacity built by firm number 1, the most senior appropriator, is certainly larger with a dam than if no dam exists. This follows because the current and future expected profits of firm number 1 are assigned first priority in the rules for operating the dam. In particular, the major costs of the dam to water users, that is, evaporation losses and interest costs as the dam is being filled, are borne by junior appropriators.

For firms junior to firm number 1 (or perhaps for firms junior to a few most senior firms), the structure of the rules for operating the dam suggest strongly that diversion capacities built with a dam are less than with no dam. Intuitively this is because with a dam, junior appropriators must bear the evaporation losses from maintaining the dam close to capacity, and must forego the use of water during the early years of dam operations when the dam is filling. We have not to derive precise analytical derivations of these intuitive results, however.

IV. ECONOMIC EFFICIENCY

Economic inefficiencies arise under the appropriative doctrine as a consequence of the unequal sharing of risk implicit in the hierarchy of priorities among otherwise identical water users. In particular we find that the appropriative system leads to production inefficiencies as aggregate expected profits are increased by the imposition of an equal sharing allocation through the redistribution of water rights. We then show that the introduction of competitive markets in which water rights may be freely bought and sold in fact effects such an equal sharing allocation, an application of the Coase theorem [4]. We first however present the following proposition, indicating the allocative ineffiency of the appropriative doctrine.

Proposition 4.1: Under the appropriative doctrine senior appropriators build more diversion capacity than do junior appropriators; i.e., $a_1 > a_2 > \ldots > a_N > 0$.

Proof: The proof is given for an arbitrary plan for releases where $\tilde{y}_t$ is the schedule for releases when t periods remain until the end of the horizon. We consider the behavior of the i^{th} and $i+1^{st}$ appropriators, from which the general result follows. The i^{th} appropriator picks a_i so as to satisfy

$$(4.1)\quad \frac{dG^t}{da_i}(R-A_{i-1},a_i) = 0 = g_2(\tilde{y}_t-A_{i-1},a_i) + \beta\frac{d}{da_i}E\{G^{t-1}[\alpha(R-\tilde{y}_t)+x-A_{i-1},a_i]\}$$

while the $i+1^{st}$ appropriator picks a_{i+1} so as to satisfy

$$(4.2)\quad \frac{dG^t}{da_i}(R-A_i,a_{i+1}) = 0 = g_2(\tilde{y}_t-A_i,a_{i+1}) + \beta\frac{d}{da_i}E\{G^{t-1}[\alpha(R-\tilde{y}_t)+x-A_i,a_{i+1}]$$

Suppose $a_i = a_{i+1}$. Since $G_{12}^{t-1}(\cdot,a_i) \geq 0$ from (3.18) and $g_{12}=0$, with $A_i > A_{i-1}$ (4.1) and (4.2) yield

$$(4.3)\quad \frac{dG^t}{da_i}(R-A_{i-1},a_i) > \frac{dG^t}{da_i}(R-A_i,a_i)$$

which is impossible. Since $G_{22}^t < 0$ we conclude that $a_i > a_{i+1}$.

Proposition 4.1 indicates a potential manifestation of inefficiency, caused by the unequal allocation of risk, but does not actually demonstrate the presence of inefficiencies. To do this we show that for any arbitrary release policy, $\tilde{y}_t$, t=1,...,T, aggregate appropriative profits are always increased by reallocating releases on the basis of equal sharing, and, that the optimal equal sharing allocation yields a global maximum.

Suppose that water rights are reassigned so that firm j receives γ_{ij}^t percent of any release in the interval $[A_{i-1},A_i]$ with t periods remaining and β_{ij}^t percent of the i^{th} firms diversion capacity a_i with t periods remaining, where $\gamma_{ij}^t \geq 0$, $\beta_{ij}^t \geq 0$, and $\sum_{j=1}^{N}\gamma_{ij}^t = 1 = \sum_{j=1}^{N}\beta_{ij}^t$, i,j=1,...,N and t = 1,...,T. For convenience let $\gamma^t = [\gamma_{ij}^t]$, $\beta^t = [\beta_{ij}^t]$ and define $\hat{a}_j^t$ by $\sum_{i=1}^{N}\beta_{ij}^t a_i = \hat{a}_j^t$. With one period remaining, aggregate profits from such a reallocation, in view of (3.1) are

$$(4.4)\quad h^1(\gamma^1,\beta^1) = \sum_{j=1}^{N} g[\sum_{k=1}^{i-1} \gamma^1_{kj} a_k + \gamma^1_{ij}(\hat{y}_1 - A_{i-1}), \hat{a}^1_j] \quad \text{if } A_{i-1} \leq \hat{y}_1 \leq A_i$$

let $H^1(R/N,\hat{a}^1) = \max_{\gamma^1,\beta^1} h^1(\gamma,\beta)$ subject to the restrictions on γ^1 and β^1, where $\hat{a}^1 = (\hat{a}^1_1,\ldots,\hat{a}^1_N)$. By the concavity of g it is clear that a maximum occurs at $\gamma^1 = [1/N] = \beta^1$ so that

$$(4.5)\quad H^1(R/N,\hat{a}^1) = Ng(\tilde{y}_1/N,a)$$

where $a = A_N/N$, that is, the optimal allocation rights is equal sharing.

If R is stochastic, it can be shown (see [3]) that $E\{H^1(R/N,\hat{a}^1)\}$ is largest when $\gamma^1 = \beta^1 = [1/N]$ also. With two periods remaining, consider the choice of γ^2,β^2 for an arbitrary $\tilde{y}_2$.

$$(4.5a)\quad h^2(\gamma^2,\beta^2) = \sum_{i=1}^{N} g[\sum_{k=1}^{i-1}\gamma^2_{kj}\, a_k + \gamma^2_{ij}(\tilde{y}_2 - A_{i-1}),\hat{a}^2_j]$$
$$+ \beta E\left\{H^1(\alpha\frac{(R-y_2)}{N} + x,\hat{a}^1)\right\} \text{ if } A_{i-1} \leq y_2 \leq A_i.$$

Since $\gamma^1,\beta^1 = [1/N]$ maximizes $h^1(\gamma^1,\beta^1)$ subject to $\sum_{j=1}^{N}\gamma_{ij} = 1$, $\sum_{j=1}^{N}\beta_{ij} = 1$, $i = 1,\ldots,N$, it follows that at $\gamma^1 = \beta^1 = [1/N]$, $(\tilde{y}_1 - A_{i-1})g_1(\cdot) + \lambda^1_i = 0$ for every $j = 1,\ldots N$, where λ^1_i is the Lagrange multiplier associated with $\sum_{j=1}^{N}\gamma^1_{ij} = 1$. Since this holds for arbitrary $\tilde{y}_1$, this implies in turn that $(\tilde{y}_2 - A_{i-1})g_1(\cdot) + \lambda^2_i = 0$ at $\gamma^2 = \beta^2 = [1/N]$, where λ^2_i is the Lagrange multiplier associated $\sum_{j=1}^{N}\gamma^2_{ij} = 1$. Since $\frac{\partial y_2}{\partial\gamma^2_{ij}} = 0$, it follows that the optimal choice of $\gamma^2 = [1/N]$. By induction it follows that $\gamma^t = [1/N] = \beta^t$ and for

$$(4.6)\quad h^t(\gamma^t,\beta^t) = \sum_{i=1}^{N} g[\sum_{k=1}^{i-1}\gamma^t_{kj}a_k + \gamma^t_{ij}(\tilde{y}_t - A_{i-1}),\ \hat{a}^t_j]$$
$$+\beta E\{H^{t-1}(\frac{\alpha(R-\tilde{y}_t)+x}{N},\hat{a}^1)\} \quad \text{if } A_{i-1} \leq \tilde{y}_t \leq A_i$$

we have

$$(4.7)\quad H^t(R/N,\hat{a}^1) = N\bar{\Pi}^t(\tilde{y}_t/N,a)$$

where

$$(4.8)\quad \Pi^t(\tilde{y}_t/N,a) \equiv \Pi^t(R/N,a)$$

with Π^t defined by (2.18). Since generally

$$(4.9)\quad \bar{\Pi}^t(\tilde{y}_t/N,a) \leq \Pi^t(R/N,a)$$

we have:

<u>Proposition 4.2</u>: For any arbitrary release policy and fixed aggregate diversion capacity by a fixed number of firms, aggregate expected profits are less under the appropriative doctrine than under the equal sharing doctrine.

<u>Corollary</u>: If reservoir capacity, aggregate diversion capacity and the number of firms are fixed, then the equal sharing doctrine and the release policy given by (2.17) yield a global optimum of aggregate expected profits.

Proposition 4.2 asserts the Pareto superiority of the doctrine of equal sharing over the appropriative doctrine (assuming compensation is paid) when stationary state conditions hold with respect to reservoir capacity, the number of firms and the aggregate investment in diversion facilities. The corollary establishes the Pareto optimality of the equal sharing allocation under similar conditions.

As allocative inefficiency in the appropriative system arises from the unequal distribution of risk among rights holders, the Coase theorem suggests that the introduction of competitive markets in water rights and diversion capacity should provide a solution to the problem. We let γ^t and β^t be as above and define p_i^t and q_i^t as prices of a one percent share of firm i's water right and diversion capacity, respectively, in period t. With the initial vector of investment, $(a_1, a_2, \ldots a_N)$, given any fixed aggregate investment, A_N, with one period remaining, firm j chooses the vectors $\gamma_j^1 = (\gamma_{1j}^1, \ldots, \gamma_{Nj}^1)$ and $\beta_j^1 = (\beta_{1j}^1, \ldots, \beta_{Nj}^1)$ so as to maximize

$$(4.10) \quad h^1(\gamma_j^1, \beta_j^1) = g[\sum_{k=1}^{i-1} \gamma_{kj}^1 a_k + \gamma_{ij}^1 (\tilde{y}_1 - A_{i-1}), \hat{a}_j^1] +$$

$$+ p_j^1 - \sum_{i=1}^{N} \gamma_{ij}^1 p_i^1 + q_j^1 - \sum_{i=1}^{N} \beta_{ij}^1 q_i^1 \quad \text{if } A_{i-1} \le \tilde{y}_1 \le A_i$$

where $\tilde{y}_t, t=1,\ldots,T$, is a fixed but arbitrary release policy. Necessary conditions require

$$(4.11) \quad p_k^1(\tilde{y}_1) = \begin{cases} g_1[\cdot, \hat{a}_j^1] a_k, & k=1,\ldots,i-1 \\ g_1[\cdot, \hat{a}_j^1](\tilde{y}_1 - A_{i-1}) & k = i \\ 0 & k = i+1,\ldots,N \end{cases}$$

if $A_{i-1} \le \tilde{y}_1 \le A_i$, and

$$(4.12) \quad q_k^1 = g_2[\cdot, \hat{a}_j^1] a_k \quad , k = 1,\ldots,N$$

In equilibrium $\sum_{j=1}^{N} \beta_j^1 = (1,1,\ldots,1) = \sum_{j=1}^{N} \gamma_j^1$ and (4.11) and (4.12) hold for $j=1,\ldots,N$. In view of the concavity of g this yields $\gamma^1 = [1/N] = \beta^1$

so that (4.11) and (4.12) become

$$(4.13) \quad p_k^1(\tilde{y}_1) = \begin{cases} g_1(\tilde{y}_1/N, a) a_k, k=1,\ldots,i-1 \\ g_1(\tilde{y}_1/N, a)(\tilde{y}_1 - A_{i-1}) \;, k=i \\ 0, \quad k = i+1,\ldots,N \end{cases}$$

if $A_{i-1} \le \tilde{y}_1 \le A_i$ and

$$(4.14) \quad q_k^1 = g_2(\tilde{y}_1/N, a) a_k \;, \; k = 1,\ldots,N.$$

Thus in fact the competitive market induces an equal sharing allocation. By analogy with the analysis leading to Proposition 4.2 we see that

$$(4.15) \quad p_k^t(\tilde{y}_t) = \begin{cases} g_1(\tilde{y}_t/N, a) a_k & k=1,\ldots,i-1 \\ g_1(\tilde{y}_t/N, a)(\tilde{y}_t - A_{i-1}), & k=i \\ 0 \;, & k = i+1,\ldots,N \end{cases}$$

if $A_{i-1} \le \tilde{y}_t \le A_i$, and

$$(4.16) \quad q_k^t = g_2(\tilde{y}_t/N, a) a_k \;, \; k=1,\ldots,N.$$

Equations (4.15) and (4.16) define the t^{th} period prices of water and diversion capacity for a given level of announced releases. For a given level of releases, these prices do not vary intertemporally. The price of one percent of a senior firm's capacity exceeds that for one percent of a junior firm's capacity as senior firms own more capacity (Proposition 4.1); however the price per unit of capacity is equalized among firms; i.e., while $r < s$ implies $q_r^t > q_s^t$, we still have $q_r^t/a_r = q_s^t/a_s$. Observe also that the separability of $\pi \equiv g$ implies that q_r^t are constant intertemporally regardless of release levels.

On the other hand (4.15) and Proposition 4.1 imply that $p_r^t \ge p_s^t$ for $r < s$ with strict inequality for $r \le i$. Since $\tilde{y}_t$ is observed (announced) before prices are determined, risk does not enter and $p^t_{i-1}/a_r = p^t_s/a_s$ for $r < s < i$, and $p^t_{i-1}/a_{i-1} = p^t_i/(\tilde{y}_t - A_{i-1})$ so that the price per unit of water is constant among suppliers.

Unfortunately there are some problems with these conclusions. First, many firms are leasing diversion capacity that they know with certainty they will not be able to use, suggesting a likely breakdown of the equilibrium attained. Secondly, (perhaps costly) transactions are required every time period, leading to uncertainty concerning the tenure of the new rights obtained and the likely unraveling of the equilibrium allocation. Consequently a more interesting question concerns the existence of contingency prices which allow rights owners to insure against low water availability by a once and for all transaction. In this case $\alpha = [\gamma_{ij}]$ and $\beta = [\beta_{ij}]$ are a priori constant between time periods; once chosen they are contractually fixed. If the contract is effected with only one period remaining, (4.13) and (4.14) define the contingency prices of water and diversion capacity. To see this let

$$(4.17)\quad h^1(\gamma_j,\beta_j) = g[\sum_{k=1}^{i-1} \gamma_{kj}a_k + \gamma_{ij}(\tilde{y}_1 - A_{i-1}), \hat{a}_j]$$

represent gross profits to firm j when $A_{i-1} \leq \tilde{y}_1 \leq A_i$. Firm j chooses γ_j and β_j so as to maximize net profits, given by

$$(4.18)\quad \bar{h}^1(\gamma_j,\beta_j;\tilde{y}_1) = h^1(\gamma_j,\beta_j) + \hat{p}^1_j - \sum_{i-1}^{N} \gamma_{ij}\hat{p}^1_i + \hat{q}^1_j - \sum_{i-1}^{N} \beta_{ij}\hat{q}^1_i$$

where $\hat{p}^1_j$ and $\hat{q}^1_j$ are the one period contingency prices of firm j's water and diversion capacity when $A_{i-1} \leq \tilde{y}_1 \leq A_i$. In view of (4.10)

and (4.11), we have

$$(4.19)\quad \hat{p}^1_k(\tilde{y}_1) = p^1_k(\tilde{y}_1)$$

with the equal sharing allocation obtaining.

With two periods remaining gross profits for firm j are

$$(4.20)\quad h^2(\gamma_j,\beta_j;\tilde{y}_2) = g[\sum_{k=1}^{i-1} \gamma_{kj}a_k + \gamma_{ij}(\tilde{y}_2 - A_{i-1}), \hat{a}^1_j] + \beta E\{h^1(\gamma_j,\beta_j;\tilde{y}_1)\}$$

when $A_{i-1} \leq \tilde{y}_2 \leq A_i$. Firm j chooses γ_j and β_j so as to maximize net profits, given by

$$(4.21)\quad \bar{h}^2(\gamma_j,\beta_j;\tilde{y}_2) = h^2(\gamma_j,\beta_j;\tilde{y}_2) + p^2_j - \sum_{i=1}^{N} \gamma_{ij}\hat{p}^2_i + \hat{q}^2_j - \sum_{i=1}^{N} \beta_{ij}\hat{q}^2_i$$

In view of the concavity of g, necessary conditions for profit maximization holding for $j = 1,\ldots,N$ imply that

$$(4.22)\quad \hat{p}^2_k(\tilde{y}_2) = p^2_k(\tilde{y}_2) + \beta E\{p^1_k(\tilde{y}_1)\}$$

and

$$(4.23)\quad \hat{q}^2_k = q^2_k + \beta q^1_k = q_k(1+\beta)$$

the last equality in (4.23) holding since the q^t_k are intertemporally constant. By induction we have generally

$$(4.24)\quad \hat{p}^t_k(\tilde{y}_t) = \sum_{s=o}^{t} \beta^t E\{p^{t-s}_k(\tilde{y}_{t-s})\}$$

where $p_k(\tilde{y}_0) \equiv 0$, and

$$(4.25)\quad \hat{q}_k^t = q_k \sum_{s=o}^{t-1} \beta^s.$$

We summarize these results as

<u>Proposition 4.3</u>: Suppose that the release policy for a reservoir is $\hat{y}_t, t=1,\dots,T$. If, under the appropriative doctrine of water rights, competitive markets in water rights and diversion capacity are introduced with transactions consummated in each time period after the actual release in that period is known, then with t periods remaining the equilibrium prices of water and diversion capacity for firm k are given by

$$(4.15')\quad p_k^t(\tilde{y}_t) = \begin{cases} \pi_1(\tilde{y}_t/N,a)a_k \ , & k=1,\dots,i-1 \\ \pi_1(\tilde{y}_t/N,a)(y_t-A_{i-1}), & k=i \\ 0 & k = i+1,\dots,N \end{cases}$$

if $A_{i-1} \leq \tilde{y}_t \leq A_i$, and

$$(4.16')\quad q_k(\tilde{y}_t) = \pi_2(\tilde{y}_t/N,a)a_k, \ k=1,\dots,N$$

with $q_k(\tilde{y}_t) = q_k$ if $\pi_{12} = 0$. These prices induce an equal sharing allocation of water and diversion capacity; moreover, the price per unit of water is equal among suppliers and the price per unit of diversion capacity is equalized among all firms. The price per unit of water right is non-decreasing with increasing seniority.

<u>Corollary</u>: If recontracting is not permitted in each time period and if the market is established with t periods remaining, the contingency prices for firm k's water $(\hat{p}_k^t)$ and diversion capacity $(\hat{q}_k^t)$ when $A_{i-1} \leq \tilde{y}_t \leq A_i$,

are given by (4.24) and (4.25) respectively. Moreover, $\hat{p}_k^t > \hat{p}_m^t$ and $\hat{p}_k^t/a_k > \hat{p}_m^t/a_m$ for $k < m$ while $\hat{q}_k^t/a_k = \hat{q}_m^t/a_m$ for all k, m.

V. DECENTRALIZED DECISION MAKING UNDER EQUAL SHARING

Propositions 2.1 and 2.2 suggest the possibility of achieving a simultaneously optimal combination of diversion capacity and release policies given independent and uncoordinated optimization decisions by firm owners and the reservoir manager. The failure of such a proposition to hold could result in informational problems or provide incentives for firms to misrepresent their intended actions and ultimately preclude the attainment of the "optimum optimorem". However, we show that such a decentralization mechanism does exist and the sequential reactions of firms to release policies (and of the reservoir manager to diversion capacity decisions) ultimately results in convergence to a simultaneously optimal steady-state combination of release policies and diversion capacities, ignoring gaming complications. The analysis is presented in the context of an infinite lived dam as this allows us to avoid increasingly cumbersome induction proofs and provides a more natural framework in which to pursue other questions in the next section.

Since at any juncture there are an infinite number of periods remaining, timescripts are redundant and hence are omitted. We let

$$(5.1)\quad \pi(y,a,R) = \pi(y/N,a) + \beta E\{\Pi[\frac{\alpha(R-y)+x}{N}, a]\}$$

represent expected profits when y is released in the current period, given that water available is R, and decisions are made optimally in periods thereafter.

Letting

$$(5.2)\quad \Pi(R/N,a) = \max_{0\le R-y\le \bar{X}} \{\pi(y/N,a)+\beta E\{\Pi[\frac{\alpha(R-y)+x}{N},a]\}\}$$

we determine the optimal release policy to be $\hat{y}$ where

$$(5.3)\quad \hat{y} = \begin{cases} R & \text{if } R \le y^* \\ \bar{y}(R) & \text{if } y^* \le R \le y^{**}+\bar{X} \\ R-\bar{X} & \text{if } R \ge y^{**}+\bar{X} \end{cases}$$

and, differentiating (5.1) with respect to y and equating the result to zero, $\bar{y}(R)$ is given by

$$(5.4)\quad \pi_1(\bar{y}/N) - \alpha\beta E\{\Pi_1[\frac{\alpha(R-\bar{y}) + x}{N},a]\} = 0$$

y* is given by the smallest value of y that satisfies

$$(5.5)\quad \pi_1(y^*/N) - \alpha\beta E\{\Pi_1(x/N,a)\} = 0$$

and y** is given by the smallest value of y that satisfies

$$(5.6)\quad \pi_1(y^{**}/N) - \alpha\beta E\{\Pi_1(\frac{\alpha\bar{X}+x}{N},a)\} = 0.^*$$

Thus

$$(5.7)\quad \Pi(R/N,a) = \begin{cases} \pi(R/N,a) + \beta E\{\Pi(x/N,a)\} & \text{if } R \le y^* \\ \pi(\bar{y}/N,a) + \beta E\{\Pi[\frac{\alpha(R-\bar{y})+x}{N},a]\} & \text{if } y^* \le R \le y^{**}+\bar{X} \\ \pi(\frac{R-\bar{X}}{N},a) + \beta E\{\Pi(\frac{\alpha\bar{X}+x}{N},a)\} & \text{if } R \ge y^{**}+\bar{X} \end{cases}$$

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* If (5.5) does not hold, then y* = a, and similarly for (5.6).

represents the profits accruing to a typical firm.* Thus we have:

Proposition 5.1: Under the equal sharing doctrine, if the manager of a reservoir with an infinite lived dam releases water so as to maximize the expected profits of N identical water users, the optimal release policy exists and is unique and is given by (5.3) with the expected profits for a typical firm given by (5.7).

Proposition 5.1 is the infinite horizon analogue to Proposition 2.1. We can establish the analogue to Proposition 2.2 in a similar manner. Differentiating (5.7) with respect to a, we have

$$(5.8)\quad \Pi_2(R/N,a) = \begin{cases} -c+\beta\frac{d}{da}E\{\Pi(x/N,a)\} & \text{if } R \le y^* \\ -c+\beta\frac{d}{da}E\{\Pi[\frac{\alpha(R-\bar{y})+x}{N},a]\} & \text{if } y^* < R \le y^{**}+\bar{X} \\ -c+\beta\frac{d}{da}E\{\Pi(\frac{\alpha\bar{X}+x}{N},a)\} & \text{if } R > y^{**}+\bar{X} \end{cases}$$

where $\Pi_2(R/N,a) = 0$ determines optimal diversion capacity. By the

*Sufficient conditions for the existence and uniqueness of $\Pi(\cdot)$ are given in Denardo [5]. However the Blackwell [2] sufficient conditions are more tractable: let the transformation T, be defined by

$$T_\Pi(R) = \max_{0\le R-y\le \bar{X}} \{\pi(y/N,a)+\beta E\{\Pi[\frac{\alpha(R-y)+x}{N},a]\}\}; \text{ if}$$

(i) T is monotone; i.e., $u(z)\ge v(z)$ implies $T_u(z)\ge T_v(z)$ for all z; and
(ii) $T_{(\Pi+a)}\le T_\Pi+\beta a$, where a is a constant, $0\le\beta<1$, and $\Pi+a = \Pi(R)+a$,

then Π is a unique fixed point; i.e., the solution to the dynamic programming problem exists and is unique. The reader can easily verify that these conditions are in fact satisfied, under the condition of the footnote of the previous page.

With somewhat more diligence one can show that $\Pi^t\to\Pi$ uniformly, which, with related demonstrations, allows us to infer that the properties of the Π^t's from the finite horizon problem obtain for Π as well (see Bellman [1]).

uniform convergence of the sequence $\{\Pi^t\}$ to Π (see footnote on p. 48), $\Pi_{12}(R/N,a) \ge 0$ while $\Pi_{12}(x/N,a) = 0$ so that $\Pi_2(R/N,a) \ge \Pi_2(x/N,a)$. As $\Pi_{22} < 0$, we have

Proposition 5.2: Under equal sharing of water among N identical expected profit maximizing firms, water users find it optimal to construct greater diversion capacity when an infinite lived reservoir is operated optimally, in the sense of Proposition 5.1, than when no dam exists.

Propositions 5.1 and 5.2 raise the question of whether the independent and uncoordinated optimizing behavior of firm owners and the reservoir manager would lead to a jointly optimal release policy and aggregate diversion capacity investment (for a fixed number of firms). We will show that if such an optimum exists, then it is attainable under decentralized behavior.

Recalling equation (5.1) and letting $\partial\pi(y,a,R)/\partial y = \pi_y$, we have* optimally

$$(5.9)\quad \pi_y = 0 = \pi_1 - \alpha\beta E\{\Pi_1\},$$

where $\pi_1 = \frac{\partial\pi}{\partial y/N}$, $\Pi_1 = \frac{\partial\Pi}{\partial z}$, with $z = \frac{\alpha(R-y)+x}{N}$.

Considering y as a function of a, differentiating (5.9) with respect to a and solving for dy/da yields

*In order to simplify some forthcoming expressions, we drop the arguments of π,Π and their derivative forms.

$$(5.10)\quad \frac{dy}{da}\Big|_{\pi_y=0} = \frac{N\alpha\beta E\{\Pi_{12}\}}{\pi_{11}+\alpha^2\beta E\{\Pi_{11}\}} < 0$$

Similarly, letting $\partial\pi(y,a,R)/\partial a = \pi_a$, we have optimally

$$(5.11)\quad \pi_a = 0 = \pi_2 + \beta \frac{d}{da}\{E\Pi\}$$

Again considering y as a function of a, differentiating (5.11) with respect to a and solving for dy/da yields

$$(5.12)\quad \frac{dy}{da}\Big|_{\pi_a=0} = \frac{N[\pi_{22}+\beta E\{\Pi_{22}\}]}{\alpha\beta E\{\Pi_{12}\}} < 0$$

If a simultaneously optimum combination $(\tilde{y},\tilde{a},)$ exists, then at a regular maximum second order conditions require that

$$(5.13)\quad \begin{vmatrix} E\{\Pi_{11}\} & E\{\Pi_{12}\} \\ E\{\Pi_{21}\} & E\{\Pi_{22}\} \end{vmatrix} > 0$$

for (y,a) close to $(\hat{y},\hat{a})$. In view of (5.13) one quickly obtains

$$(5.14)\quad \left| \frac{dy}{da}\Big|_{\pi_y=0} \right| < \left| \frac{dy}{da}\Big|_{\pi_a=0} \right|$$

which is sufficient for the desired convergence. Thus we have:

<u>Proposition 5.3</u>: Under equal sharing of water released from an infinite lived dam, suppose there exists an optimal* combination $(\tilde{y},\tilde{a})$

such that $\tilde{y}$ maximizes expected profits given $\tilde{a}$, and $\tilde{a}$ maximizes expected profits given $\tilde{y}$. Then assuming a regular maximum, $(\tilde{y},\tilde{a})$ is attainable through the independent and uncoordinated optimizing behavior of the reservoir manager and the N firms. (See Figure 5.1)

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*In view of (5.10) and (5.12), $(\tilde{y},\tilde{a})$ is unique and the proposition holds globally.

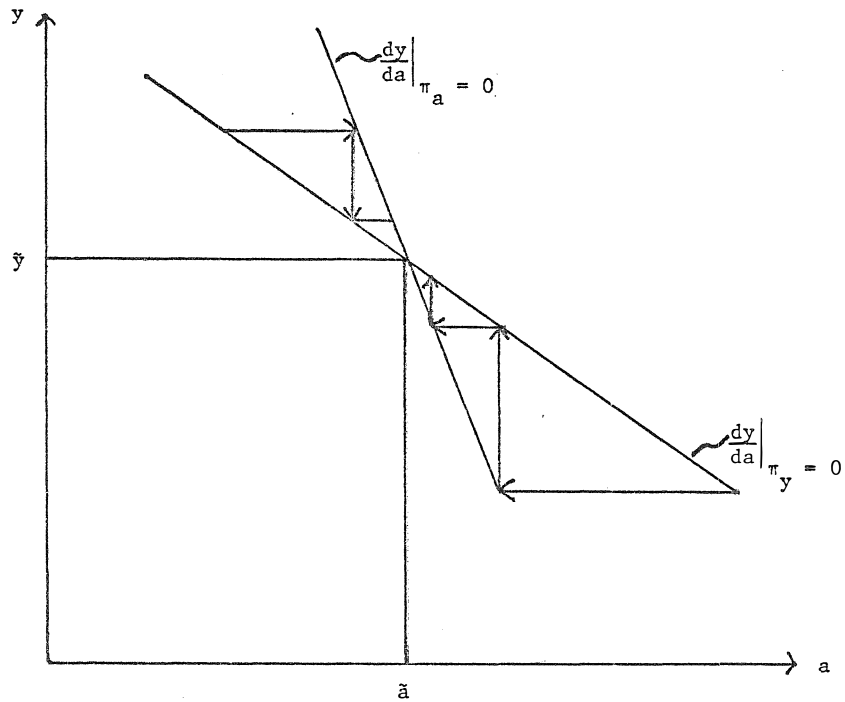

Figure 5.1: Decentralization Mechanism Under Equal Sharing

VI. OTHER RESULTS

In this section we adduce some results concerning the relationships between investment in reservoir capacity and investment in diversion capacity under the doctrine of equal sharing.* We maintain the assumption concerning the fixity of N in order to fix ideas. Where aggregate investment in diversion capacity is relevant we allow it to change only through the alteration of capacities by individual firms and not through industry exit or entry. Alternatively we could fix aggregate investment in diversion capacity and search for optimum number of firms. The ramifications of unrestricted entry and exit with regard to optimal firm size and number in a static setting are explored in [3]. We have found no reason to extrapolate those results to a dynamic environment and are content to observe that the possibility of incentive incompatibility arises under the doctrine of equal sharing, but this may, however, be precluded by the presence of a range of decreasing average costs in the construction of diversion capacity.

Our first result concerns the optimal size of the reservoir. We propose the fiction that downstream users of water pay the costs of dam building and maintenance, although this assumption can be modified with no substantive alteration of results. For symmetry we assume

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*As, under the appropriative doctrine, the aggregate profit function may be non-concave, some of these results do not translate for appropriative water rights. Moreover, the inefficiencies inherent in the appropriative system renders the analogous results uninteresting, regardless.

that $\beta = 1/(1+r)$ where r is the market rate of interest. If $C(\bar{X})$ is the cost of constructing (plus perhaps the present value of operating and maintaining) a dam of size $\bar{X}$ and owners make fixed payment at the end of each time period, it follows that this annual payment for the representative firm is

$$(6.1)\quad \frac{rC(\bar{X})}{N} = \frac{(1-\beta)C(\bar{X})}{\beta N}.$$

Letting $\bar{\pi}(y,a,R)$ be net profits accruing to the typical firm when dam size is $\bar{X}$, releases are y, diversion capacity is a, water available is R and decisions are to be made optimally in all remaining time periods, we have

$$(6.2)\quad \bar{\pi}(y,a,R) = \pi(y/N,a) - \beta rC(\bar{X})/N + \beta E\{\bar{\Pi}[\frac{\alpha(R-y)+x}{N},a]\}$$

where $\bar{\Pi}(R/N,a) = \max_{0\le R-y\le \bar{X}} \bar{\pi}(y,a,R)$ so that

$$(6.3)\quad \bar{\Pi}(R/N,a) = \begin{cases} \pi(R/N,a)-\beta rC(\bar{X})/N+\beta E\{\bar{\Pi}(x,N,a)\} & \text{if } R \le y^* \\ \pi(\bar{y}/N,a)-\beta rC(\bar{X})/N+\beta E\{\bar{\Pi}[\frac{\alpha(R-\bar{y})+x}{N},a]\} & \text{if } y^*\le R \le y^{**}+\bar{X} \\ \pi(\frac{R-\bar{X}}{N},a)-\beta rC(\bar{X})/N+\beta E\{\bar{\Pi}(\frac{\alpha\bar{X}+x}{N},a)\} & \text{if } R \ge y^{**} + \bar{X} \end{cases}$$

where y^*, y^{**} and $\bar{y}(R)$ are as defined by (5.5), (5.6) and (5.4). Letting $\partial\bar{\Pi}/\partial\bar{X} \equiv \bar{\Pi}_{\bar{X}}$, optimal dam size is determined by setting $\bar{\Pi}_{\bar{X}} = 0$. Performing the necessary differentiation, with $dC/d\bar{X} = C'(\bar{X})$, we have

$$(6.4)\quad \bar{\Pi}_{\bar{X}}(R/N,a) = \begin{cases} -\beta rC'(\bar{X})/N & \text{if } R \le y^* \\ -\beta rC'(\bar{X})/N & \text{if } y^* \le R \le y^{**} + \bar{X} \\ \{\alpha\beta E\{\bar{\Pi}_1(\frac{\alpha\bar{X}+x}{N},a)\}-\pi_1(\frac{R-\bar{X}}{N})-\beta rC'(\bar{X})\}/N & \text{if } R\ge y^{**}+\bar{X} \end{cases}$$

so that the third line of (6.4) defines the optimal value of $\bar{X}$, say $\bar{X}^*$. Thus at the margin, the discounted (for time distance and evaporation loss) expected gains from having a larger dam and storing more water in the event of high run-off (i.e., a smaller dam would mean (greater) water spills) must equal the marginal annualized construction and O-M costs plus the marginal opportunity cost of not using additionally stored water in the current period. Since water users pay all costs, we must append (6.4) with the condition

$$(6.5)\quad \bar{\Pi}(R/N,a)|_{\bar{X}=\bar{X}^*} \ge 0$$

A related question concerns the effect of increased investment in reservoir capacity on the optimal diversion capacity for firms. Differentiating (6.4) with respect to a and solving for $\partial a/\partial\bar{X}$, we find that

$$(6.6)\quad \frac{\partial a}{\partial \bar{X}} = \begin{cases} 0 & \text{if } R \le y^{**} + \bar{X} \\ \dfrac{N\alpha\beta E\{\bar{\Pi}_{12}(\frac{\alpha\bar{X}+x}{N},a)\}}{\alpha^2\beta E\{\bar{\Pi}_{11}(\frac{\alpha\bar{X}+x}{N},a)\} + \pi_{11}(\frac{R-\bar{X}}{N})-\beta rNC''(\bar{X})} & \text{if } R \ge y^{**} + \bar{X} \end{cases}$$

Since $\partial a/\partial\bar{X}\le 0$ investment in reservoir capacity and investment in diversion capacity are substitutes rather than complements, as might have been thought. Also from equation (5.6) we can easily verify that

$$(6.7)\quad \frac{\partial y^{**}}{\partial \bar{X}} = \alpha^2 \beta E\{\bar{\Pi}_{11}(\frac{\alpha\bar{X}+x}{N},a)\}/\pi_{11}(y_1^{**}/N) > 0$$

as might be expected.

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